MARK SCHEME for the May/June 2012 question paper for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2, 1, 0 means that the candidate can earn anything from 0 to 2.
The following abbreviations may be used in a mark scheme or used on the scripts:

**AG**  Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

**BOD**  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

**CAO**  Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

**ISW**  Ignore Subsequent Working

**MR**  Misread

**PA**  Premature Approximation (resulting in basically correct work that is insufficiently accurate)

**SOS**  See Other Solution (the candidate makes a better attempt at the same question)

### Penalties

- **MR −1**  A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

- **OW −1,2**  This is deducted from A or B marks when essential working is omitted.

- **PA −1**  This is deducted from A or B marks in the case of premature approximation.

- **S −1**  Occasionally used for persistent slackness – usually discussed at a meeting.

- **EX −1**  Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.
### Question 1

1. (i) 

![Graph](image)

B1, B1

B1 for shape

B1 for both intercepts

(ii) $2x - 5 = \pm 3$, leading to $x = 1, 4$

B1, B1

[4]

### Question 2

2. $f(-2) : 4a - 2b = 46$

$f\left(\frac{1}{2}\right) : a + 2b = -21$

$a = 5, b = -13$

M1

M1 for substitution of $x = -2$ and equating to zero

M1 for substitution of $x = 0.5$ and equating to $-35$

M1 for solution of equations

### Question 3

3. $x^2 + x(k-2) + (5-k) = 0$

Using $b^2 > 4ac$, $(k-2)^2 > 4(5-k)$

$k^2 > 16$

$k > 4, k < -4$

M1

DM1

A1

A1

A1, A1

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M1 for equating line and curve

DM1 for use of $b^2 > 4ac$

b = $k-2$ and c = $5-k$

Accept $\leq$ etc.

A1 for each

### Question 4

4. (a) (i) 15120

B1

B1

(ii) 210

B1

(b) (i) 15504

B1

(ii) $^{12}\text{C}_{10} \times ^{8}\text{C}_5 = 3696$

B1, B1

B1

(iii) 56

B1

[7]

### Question 5

5. (i) $(0, 4)$

$\frac{dy}{dx} = 3x^2 + 4x - 3$

When $x = 0$, $\frac{dy}{dx} = -3$

$y - 4 = -3x$

M1

M1 for differentiation

M1 for attempt at line equation

A1

A1 need $x$ and $y$

(ii) $4 - 3x = x^3 + 2x^2 - 3x + 4$

leading to, $0 = x^3 + 2x^2$

$(-2, 10)$

M1

M1

M1

M1 for equating line and curve

M1 for solution of cubic

A1

A1 need $x$ and $y$
6 (i) \[ 15 + 2\sin^2 \theta = \frac{7}{\cos^2 \theta} \]
\[ 15 + 2\tan^2 \theta = 7\sec^2 \theta \]
\[ 15 + 2\tan^2 \theta = 7(1 + \tan^2 \theta) \]
leading to \[ \tan^2 \theta = \frac{8}{5} \]
or
\[ 15\cos^2 \theta + 2\sin \theta = 7(\cos^2 \theta + \sin^2 \theta) \]
\[ 8\cos^2 \theta = 5\sin^2 \theta \]
leading to \[ \tan^2 \theta = \frac{8}{5} \]

(ii) \[ \tan \theta = \pm \frac{8}{\sqrt{5}} \]
leading to \[ \theta = 0.902, 2.24 \]
(also, \[ \sin \theta = \pm \frac{8}{\sqrt{13}}, \cos \theta = \pm \frac{5}{\sqrt{13}} \])

7 (i) \[ \frac{y}{x} = A + Bx \]

(ii) Grad = \( B = -0.5 \)
Intercept = \( A = 3 \)

8 (a) \[ 2\lg x - \lg(5x + 60) = \lg 10 \]
\[ \frac{x^2}{5x + 60} = 10 \]
leading to \[ x^2 - 50x - 600 = 0 \]
x = 60

(b) \[ \log_5 y = \frac{4\log_5 5}{\log_5 y} \]
\[ (\log_5 y)^2 = 4 \]
\[ \log_5 y = \pm 2 \]
y = 25, \frac{1}{25}
9 \[120(p^7q^3) \text{ and } (45)p^8q^2\]
\[120p^7q^3 = 270p^8q^2\]
\[252p^5q^5 = 252\]
\[pq = 1 \text{ and } 4q = 9p\]
leading to \(p = \frac{2}{3}, q = \frac{3}{2}\)

- B3,2,1,0 \(-1 \text{ per element (of 4) incorrect}\)
- M1 \- M1 for equating and multiplying by 6
- B1 \- B1 for \(252p^5q^5\)
- B1 \- B1 for \(pq = 1\) OR \(4p = 9q\)
- A1, A1 \- A1 for each

10 (i) \[
\frac{dy}{dx} = 2e^{2x} - 2e^{-2x}
\]

- B1, B1 \- One per term

(ii) \[3 = 2e^{2x} - 2e^{-2x}\]
\[2e^{4x} - 3e^{2x} - 2 = 0\]
\[(2e^{2x} + 1)(e^{2x} - 2) = 0\]
\[e^{2x} = 2, y = \frac{5}{2}\]

- M1 \- M1 for attempt to obtain in ‘quadratic’ form
- DM1 \- DM1 for attempt to solve
- M1 \- M1 for attempt to solve for \(y\)
- A1

(iii) \[
\frac{dx}{dt} = -0.5,
\]
\[
\frac{dy}{dt} = (2e^{2x} - 2e^{-2x}) \times (-0.5)
\]
\[= -7.25\]

- M1, M1 \- M1 for substitution of \(x = 1\)
- A1 \- M1 for correct application of chain rule

11 EITHER

(i) \[
\frac{dy}{dx} = 18x - 3x^2
\]

- M1 \- M1 for differentiation

When \(\frac{dy}{dx} = 0, 0 = 3(x(6-x))\)

Turning points when \(x = 0, 6\)

When \(x = 6, y = 108\)

- M1 \- M1 for equating to zero and attempt to solve
- M1 \- M1 for finding \(y\)

(ii) Area \[
\left[ 3x^3 - \frac{x^4}{4} \right]_0^9
\]
\[= 546.75\]
\[B(0, -18)\]

Area of triangle = 81

Total Area = 628

Or: Area \[\int_0^9 (9x^2 - x^3 - 2x + 18) \, dx\]
\[\left[ 3x^3 - \frac{x^4}{4} - x^2 + 18x \right]_0^9 = 628\]

M1, A1 \- M1 for attempt to integrate

DM1, A1 \- DM1 for correct application of limits

B1 \- B1 for area of triangle

DM1, A1 \- DM1 for correct application of limits

A3,2,1,0 \- A3,2,1,0

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11 OR

(i) \( \frac{dy}{dx} = 6 \cos 3x \)

When \( x = \frac{\pi}{9} \), \( \frac{dy}{dx} = 3 \), \( y = \sqrt{3} \)

Equation of normal

\[
y - \sqrt{3} = -\frac{1}{3} \left( x - \frac{\pi}{9} \right)
\]

When \( x = 0 \), \( y = 1.85 \)

B1 for differentiation

B1 for differentiation

(ii) \( \frac{1}{2} \left( \sqrt{3} + 1.85 \right) \frac{\pi}{9} - \int_{\frac{\pi}{9}}^{\pi} 2 \sin 3x \, dx \)

\[
0.6251 - \left[ -\frac{2}{3} \cos 3x \right]_{\frac{\pi}{9}}^{\pi} = 0.292
\]

B1 for trapezium – allow unsimplified

B1 for trapezium – allow unsimplified

0.6251 - \( \left( \frac{1}{3} \right) \) = 0.292

M1 for attempt to integrate

M1 for attempt to integrate

Alt method:

Area = \( \int_{0}^{\frac{\pi}{9}} \sqrt{3} - \frac{1}{3} \left( x - \frac{\pi}{9} \right) - 2 \sin 3x \, dx \)

\[
\left[ \sqrt{3x} - \frac{x^2}{6} + \frac{\pi x}{27} + \frac{2}{3} \cos 3x \right]_{0}^{\frac{\pi}{9}} = 0.292
\]

M1, A1 for correct application of limits

M1, A1 for correct application of limits

[ DM1 ]

A1 [10]