A thin card is inserted between two separate iron cores. A coil is wound around one core as shown in Fig. 1.1.

Fig. 1.1

A current in the coil may induce an e.m.f. in another coil wound on the other core. The induced e.m.f. $V$ depends on the thickness $t$ of the card.

A student suggests that

$$V = V_0 e^{-\alpha t}$$

where $V_0$ is the induced e.m.f. without card between the cores and $\alpha$ is a constant.

Design a laboratory experiment to test the relationship between $V$ and $t$ and determine the value of $\alpha$. You should draw a diagram, on page 3, showing the arrangement of your equipment. In your account you should pay particular attention to

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken.
<table>
<thead>
<tr>
<th>Defining the problem</th>
<th>Methods of data collection</th>
<th>Method of analysis</th>
<th>Safety considerations</th>
<th>Additional detail</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A student investigates how the final velocity $v$ of a cylinder rolling down a board varies with the height $h$ of the board as shown in Fig. 2.1.

For different values of $h$, the velocity $v$ is determined using a light sensor connected to a data logger.

It is suggested that $v$ and $h$ are related by the equation

$$2gh = v^2 Z$$

where $g$ is the acceleration of free fall and $Z$ is a constant.

(a) A graph is plotted of $v^2$ on the $y$-axis against $h$ on the $x$-axis. Determine an expression for the gradient in terms of $g$ and $Z$.

gradient = ................................................. [1]
(b) Values of $h$ and $v$ are given in Fig. 2.2.

<table>
<thead>
<tr>
<th>$h$/m</th>
<th>$v$/m s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.230</td>
<td>1.40 ± 0.05</td>
</tr>
<tr>
<td>0.280</td>
<td>1.55 ± 0.05</td>
</tr>
<tr>
<td>0.320</td>
<td>1.65 ± 0.05</td>
</tr>
<tr>
<td>0.360</td>
<td>1.75 ± 0.05</td>
</tr>
<tr>
<td>0.400</td>
<td>1.85 ± 0.05</td>
</tr>
<tr>
<td>0.450</td>
<td>1.95 ± 0.05</td>
</tr>
</tbody>
</table>

Fig. 2.2

Calculate and record values of $v^2$/m$^2$ s$^{-2}$ in Fig. 2.2. Include the absolute uncertainties in $v^2$. [3]

(c) (i) Plot a graph of $v^2$/m$^2$ s$^{-2}$ against $h$/m. Include error bars for $v^2$. [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]

(iii) Determine the gradient of the line of best fit. Include the uncertainty in your answer.

gradient = ................................................. [2]
(d) The experiment is repeated with $h = 0.700\,\text{m}$.

(i) Using your answer to (c)(iii), determine the value of $v$ using the relationship given.

$v = ......................................... \text{m}\,\text{s}^{-1} \quad [1]\]

(ii) Determine the percentage uncertainty in the value of $v$.

percentage uncertainty = ............................................ % \quad [1]

(e) The constant $Z$ is given by

$$Z = \left(1 + \frac{K}{mr^2}\right)$$

where $m$ is the mass of the cylinder and $r$ is the radius of the cylinder.

Using your answers to (a) and (c)(iii), determine the value of $K$. Include the absolute uncertainty in your value and an appropriate unit.

Data: $g = 9.81\,\text{m}\,\text{s}^{-2}$, $m = 2.5\,\text{kg}$ and $r = 0.015\,\text{m}$.

$$K = ............................................... \quad [3]$$