Notation List

For Cambridge International Mathematics Qualifications

For use from 2020
Mathematical notation

Examinations for CIE syllabuses may use relevant notation from the following list.

1 Set notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\in$</td>
<td>is an element of</td>
</tr>
<tr>
<td>$\notin$</td>
<td>is not an element of</td>
</tr>
<tr>
<td>${x_1, x_2, \ldots}$</td>
<td>the set with elements $x_1, x_2, \ldots$</td>
</tr>
<tr>
<td>${x : \ldots}$</td>
<td>the set of all $x$ such that $\ldots$</td>
</tr>
<tr>
<td>$n(A)$</td>
<td>the number of elements in set $A$</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>the empty set</td>
</tr>
<tr>
<td>$\mathcal{U}$</td>
<td>the universal set</td>
</tr>
<tr>
<td>$\cup$</td>
<td>the union of the set $A$</td>
</tr>
<tr>
<td>$\mathbb{N}$</td>
<td>the set of natural numbers, ${1, 2, 3, \ldots}$</td>
</tr>
<tr>
<td>$\mathbb{Z}$</td>
<td>the set of integers, ${0, \pm 1, \pm 2, \pm 3, \ldots}$</td>
</tr>
<tr>
<td>$\mathbb{Q}$</td>
<td>the set of rational numbers, $\left{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right}$</td>
</tr>
<tr>
<td>$\mathbb{R}$</td>
<td>the set of real numbers</td>
</tr>
<tr>
<td>$\mathbb{C}$</td>
<td>the set of complex numbers</td>
</tr>
<tr>
<td>$(x, y)$</td>
<td>the ordered pair $x, y$</td>
</tr>
<tr>
<td>$\subseteq$</td>
<td>is a subset of</td>
</tr>
<tr>
<td>$\subset$</td>
<td>is a proper subset of</td>
</tr>
<tr>
<td>$\cup$</td>
<td>union</td>
</tr>
<tr>
<td>$\cap$</td>
<td>intersection</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>the closed interval ${x \in \mathbb{R} : a \leq x \leq b}$</td>
</tr>
<tr>
<td>$(a, b)$</td>
<td>the interval ${x \in \mathbb{R} : a &lt; x &lt; b}$</td>
</tr>
<tr>
<td>$(a, b]$</td>
<td>the interval ${x \in \mathbb{R} : a &lt; x \leq b}$</td>
</tr>
<tr>
<td>$(a, b)$</td>
<td>the open interval ${x \in \mathbb{R} : a &lt; x &lt; b}$</td>
</tr>
<tr>
<td>$(S, \circ)$</td>
<td>the group consisting of the set $S$ with binary operation $\circ$</td>
</tr>
</tbody>
</table>

2 Miscellaneous symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$=$</td>
<td>is equal to</td>
</tr>
<tr>
<td>$\neq$</td>
<td>is not equal to</td>
</tr>
<tr>
<td>$\equiv$</td>
<td>is identical to or is congruent to</td>
</tr>
<tr>
<td>$\approx$</td>
<td>is approximately equal to</td>
</tr>
<tr>
<td>$\sim$</td>
<td>is distributed as</td>
</tr>
<tr>
<td>$\cong$</td>
<td>is isomorphic to</td>
</tr>
<tr>
<td>$\propto$</td>
<td>is proportional to</td>
</tr>
<tr>
<td>$&lt;$</td>
<td>is less than</td>
</tr>
<tr>
<td>$\leq$</td>
<td>is less than or equal to</td>
</tr>
<tr>
<td>$\geq$</td>
<td>is greater than</td>
</tr>
<tr>
<td>$\geqslant$</td>
<td>is greater than or equal to</td>
</tr>
<tr>
<td>$\infty$</td>
<td>infinity</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>implies</td>
</tr>
<tr>
<td>$\Leftarrow$</td>
<td>is implied by</td>
</tr>
<tr>
<td>$\iff$</td>
<td>implies and is implied by (is equivalent to)</td>
</tr>
</tbody>
</table>
3 Operations

\( a + b \) \quad a \) plus \( b \\n\( a - b \) \quad a \) minus \( b \\n\( a \times b, ab \) \quad a \) multiplied by \( b \\n\( a \div b, \frac{a}{b} \) \quad a \) divided by \( b \\n\[ \sum_{i=1}^{n} a_i \] \quad \( a_1 + a_2 + \ldots + a_n \) \\n\( \sqrt[n]{a} \) \quad the \( n \)th \ root \ of \( a \), \( \text{for} \ a \in \mathbb{R}, \ a \geq 0 \) \\n\( |a| \) \quad the \ modulus \ of \( a \) \\n\( n! \) \quad \( n \) factorial \\n\( \binom{n}{r} \) \quad the \ binomial \ coefficient \ \frac{n!}{r!(n-r)!} \ for \ n, \ r \in \mathbb{Z} \ and \ 0 \leq r \leq n \\

4 Functions

\( f(x) \) \quad the \ value \ of \ the \ function \( f \) \ at \( x \) \\n\( f : A \rightarrow B \) \quad \( f \) \ is \ a \ function \ under \ which \ each \ element \ of \ set \( A \) \ has \ an \ image \ in \ set \( B \) \\n\( f : x \mapsto y \) \quad the \ function \( f \) \ maps \ the \ element \( x \) \ to \ the \ element \( y \) \\n\( f^{-1} \) \quad the \ inverse \ function \ of \ the \ one-one \ function \( f \) \\n\( gf \) \quad the \ composite \ function \ of \( f \) \ and \( g \) \ which \ is \ defined \ by \( g(f(x)) = g(f(x)) \) \\n\( \lim_{x \rightarrow a} f(x) \) \quad the \ limit \ of \( f(x) \) \ as \( x \) \ tends \ to \( a \) \\n\( \Delta x, \delta x \) \quad an \ increment \ of \( x \) \\n\( \frac{dy}{dx} \) \quad the \ derivative \ of \( y \) \ with \ respect \ to \( x \) \\n\( \frac{d^n y}{dx^n} \) \quad the \( n \)th \ derivative \ of \( y \) \ with \ respect \ to \( x \) \\n\( f'(x), f''(x), \ldots, f^{(n)}(x) \) \quad the \ first, \ second, \ \ldots, \( n \)th \ derivatives \ of \( f(x) \) \ with \ respect \ to \( x \) \\n\( \int y \, dx \) \quad the \ indefinite \ integral \ of \( y \) \ with \ respect \ to \( x \) \\n\( \int_a^b y \, dx \) \quad the \ definite \ integral \ of \( y \) \ with \ respect \ to \( x \) \ between \ the \ limits \( x = a \) \ and \( x = b \) \\n\( x, \dot{x}, \ldots \) \quad the \ first, \ second, \ \ldots \ derivatives \ of \( x \) \ with \ respect \ to \( t \) \\

5 Exponential and logarithmic functions

\( e \) \quad base \ of \ natural \ logarithms \\n\( e^x, \exp(x) \) \quad exponential \ function \ of \( x \) \\n\( \log_a x \) \quad logarithm \ to \ the \ base \( a \) \ of \( x \) \\n\( \ln x \) \quad natural \ logarithm \ of \( x \) \\n\( \log x, \log_{10} x \) \quad logarithm \ of \( x \) \ to \ base \( 10 \)
6 Circular and hyperbolic functions

\[
\begin{align*}
\sin, \cos, \tan & \quad \text{the circular functions} \\
\cosec, \sec, \cot & \quad \text{the inverse circular functions} \\
\sin^{-1}, \cos^{-1}, \tan^{-1} & \quad \text{the hyperbolic functions} \\
\cosec^{-1}, \sec^{-1}, \cot^{-1} & \quad \text{the inverse hyperbolic functions}
\end{align*}
\]

7 Complex numbers

- \(i\) the imaginary unit, \(i^2 = -1\)
- \(z\) a complex number, \(z = x + iy = r(\cos \theta + i \sin \theta)\)
- \(\text{Re } z\) the real part of \(z\), \(\text{Re } z = x\)
- \(\text{Im } z\) the imaginary part of \(z\), \(\text{Im } z = y\)
- \(|z|\) the modulus of \(z\), \(|z| = \sqrt{x^2 + y^2}\)
- \(\text{arg } z\) the argument of \(z\), \(\text{arg } z = \theta\) where \(-\pi < \theta \leq \pi\)
- \(z^*\) the complex conjugate of \(z\), \(x - iy\)

8 Matrices

- \(M\) a matrix \(M\)
- \(M^{-1}\) the inverse of the non-singular square matrix \(M\)
- \(\det M, |M|\) the determinant of the square matrix \(M\)
- \(I\) an identity (or unit) matrix

9 Vectors

- \(\mathbf{a}\) the vector \(\mathbf{a}\)
- \(\overrightarrow{AB}\) the vector represented in magnitude and direction by the directed line segment \(AB\)
- \(\hat{a}\) a unit vector in the direction of \(\mathbf{a}\)
- \(\mathbf{i}, \mathbf{j}, \mathbf{k}\) unit vectors in the directions of the Cartesian coordinate axes
- \(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\) the vectors \(xi + yj\) (in 2 dimensions) and \(xi + yj + zk\) (in 3 dimensions)
- \(|\mathbf{a}|, a\) the magnitude of \(\mathbf{a}\)
- \(\overrightarrow{AB}, |\overrightarrow{AB}|, \overrightarrow{AB}\) the magnitude of \(\overrightarrow{AB}\)
- \(\mathbf{a} \cdot \mathbf{b}\) the scalar product of \(\mathbf{a}\) and \(\mathbf{b}\)
- \(\mathbf{a} \times \mathbf{b}\) the vector product of \(\mathbf{a}\) and \(\mathbf{b}\)
10 Probability and statistics

$A, B, C, \ldots$ events

$A \cup B$ union of the events $A$ and $B$

$A \cap B$ intersection of the events $A$ and $B$

$P(A)$ probability of the event $A$

$A'$ complement of the event $A$

$P(A \mid B)$ probability of the event $A$ conditional on the event $B$

$^nC_r$ the number of combinations of $r$ objects from $n$, $^nC_r = \frac{n!}{r!(n-r)!}$

$^nP_r$ the number of permutations of $r$ objects from $n$, $^nP_r = \frac{n!}{(n-r)!}$

$X, Y, R, \ldots$ random variables

$x, y, r, \ldots$ values of the random variables $X, Y, R, \ldots$

$x_1, x_2, \ldots$ observations

$f_1, f_2, \ldots$ frequencies with which the observations $x_1, x_2, \ldots$ occur

$p(x)$ probability function $P(X = x)$ of the discrete random variable $X$

$p_1, p_2, \ldots$ probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable $X$

$f(x)$ value of the probability density function of a continuous random variable $X$

$F(x)$ value of the cumulative distribution function of a continuous random variable $X$

$E(X)$ expectation of the random variable $X$

$E(g(X))$ expectation of $g(X)$

$\text{Var}(X)$ variance of the random variable $X$

$G_X(t)$ probability generating function for the discrete random variable $X$

$M_X(t)$ moment generating function for the random variable $X$

$B(n, p)$ binomial distribution with parameters $n$ and $p$

$\text{Geo}(p)$ geometric distribution with parameter $p$

$\text{Po}(\lambda)$ Poisson distribution with parameter $\lambda$

$N(\mu, \sigma^2)$ normal distribution with mean $\mu$ and variance $\sigma^2$

$\mu$ population mean

$\sigma^2$ population variance

$\sigma$ population standard deviation

$\bar{x}$ sample mean, $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$

$s^2$ unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$\rho$ product moment correlation coefficient for a population

$r$ product moment correlation coefficient for a sample

$\phi$ probability density function of the standardised normal variable $Z \sim N(0, 1)$

$\Phi$ cumulative distribution function of the standardised normal variable $Z \sim N(0, 1)$

$H_0, H_1$ null and alternative hypotheses for a hypothesis test