A particle $P$ is projected vertically upwards with speed $20 \text{ m s}^{-1}$ from a point on the ground.

(a) Find the greatest height above the ground reached by $P$. 

(b) Find the total time from projection until $P$ returns to the ground.
A constant resistance of magnitude 1350 N acts on a car of mass 1200 kg.

(a) The car is moving along a straight level road at a constant speed of 32 m s\(^{-1}\).

Find, in kW, the rate at which the engine of the car is working. [2]

(b) The car travels at a constant speed down a hill inclined at an angle of \(\theta^\circ\) to the horizontal, where \(\sin\theta^\circ = \frac{1}{20}\), with the engine working at 31.5 kW.

Find the speed of the car. [3]
Three small smooth spheres $A$, $B$ and $C$ of equal radii and of masses 4 kg, 2 kg and 3 kg respectively, lie in that order in a straight line on a smooth horizontal plane. Initially, $B$ and $C$ are at rest and $A$ is moving towards $B$ with speed $6 \text{ m s}^{-1}$. After the collision with $B$, sphere $A$ continues to move in the same direction but with speed $2 \text{ m s}^{-1}$.

(a) Find the speed of $B$ after this collision. 

(b) Sphere $B$ collides with $C$. In this collision these two spheres coalesce to form an object $D$.

Find the speed of $D$ after this collision.
(c) Show that the total loss of kinetic energy in the system due to the two collisions is 38.4 J. [2]
A particle of mass 20 kg is on a rough plane inclined at an angle of 30° to the horizontal. A force of magnitude 25 N, acting at an angle of 20° above a line of greatest slope of the plane, is used to prevent the particle from sliding down the plane. The coefficient of friction between the particle and the plane is \( \mu \).

(a) Complete the diagram below to show all the forces acting on the particle. [1]

(b) Find the least possible value of \( \mu \). [5]
A car of mass 1200 kg is pulling a trailer of mass 800 kg up a hill inclined at an angle of \( \sin^{-1}(0.1) \) to the horizontal. The car and the trailer are connected by a light rigid tow-bar which is parallel to the road. The driving force of the car’s engine is 2500 N and the resistances to the car and trailer are 300 N and 100 N respectively.

(a) Find the acceleration of the system and the tension in the tow-bar.

[4]
(b) When the car and trailer are travelling at a speed of $30 \text{ m s}^{-1}$, the driving force becomes zero.

Find the time, in seconds, before the system comes to rest and the force in the tow-bar during this time. [5]
A particle \( P \) moves in a straight line. The velocity \( v \text{ m s}^{-1} \) at time \( t \text{ s} \) is given by

\[
\begin{align*}
    v &= 5(t - 2) \quad \text{for } 0 \leq t \leq 4, \\
    v &= k \quad \text{for } 4 \leq t \leq 14, \\
    v &= 68 - 2t \quad \text{for } 14 \leq t \leq 20,
\end{align*}
\]

where \( k \) is a constant.

(a) Find \( k \).  \[1\]

(b) Sketch the velocity–time graph for \( 0 \leq t \leq 20 \). \[3\]

(c) Find the set of values of \( t \) for which the acceleration of \( P \) is positive. \[2\]
(d) Find the total distance travelled by $P$ in the interval $0 \leq t \leq 20.$ [5]
Two particles $A$ and $B$, of masses 0.8 kg and 0.2 kg respectively, are connected by a light inextensible string. Particle $A$ is placed on a horizontal surface. The string passes over a small smooth pulley $P$ fixed at the edge of the surface, and $B$ hangs freely. The horizontal section of the string, $AP$, is of length 2.5 m (see diagram). The particles are released from rest with both sections of the string taut.

(a) Given that the surface is smooth, find the time taken for $A$ to reach the pulley. [5]
(b) It is given instead that the surface is rough and that the speed of \( A \) immediately before it reaches the pulley is \( v \) m\( \text{s}^{-1} \). The work done against friction as \( A \) moves from rest to the pulley is 2 J.

Use an energy method to find \( v \). [4]