READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
1 The standard deviation of the heights of adult males is 7.2 cm. The mean height of a sample of 200 adult males is found to be 176 cm.

(i) Calculate a 97.5% confidence interval for the mean height of adult males. [3]

(ii) State a necessary condition for the calculation in part (i) to be valid. [1]

2 A headteacher models the number of children who bring a ‘healthy’ packed lunch to school on any day by the distribution $B(150, p)$. In the past, she has found that $p = \frac{1}{3}$. Following the opening of a fast food outlet near the school, she wishes to test, at the 1% significance level, whether the value of $p$ has decreased.

(i) State the null and alternative hypotheses for this test. [1]
On a randomly chosen day she notes the number, \( N \), of children who bring a ‘healthy’ packed lunch to school. She finds that \( N = 36 \) and then, assuming that the null hypothesis is true, she calculates that \( P(N \leq 36) = 0.0084 \).

(ii) State, with a reason, the conclusion that the headteacher should draw from the test. \[2\]

(iii) According to the model, what is the largest number of children who might bring a packed lunch to school? \[1\]

3 A population has mean 12 and standard deviation 2.5. A large random sample of size \( n \) is chosen from this population and the sample mean is denoted by \( \bar{X} \). Given that \( P(\bar{X} < 12.2) = 0.975 \), correct to 3 significant figures, find the value of \( n \). \[4\]
Small drops of two liquids, $A$ and $B$, are randomly and independently distributed in the air. The average numbers of drops of $A$ and $B$ per cubic centimetre of air are 0.25 and 0.36 respectively.

(i) A sample of 10 cm$^3$ of air is taken at random. Find the probability that the total number of drops of $A$ and $B$ in this sample is at least 4.
(ii) A sample of 100 cm$^3$ of air is taken at random. Use an approximating distribution to find the probability that the total number of drops of $A$ and $B$ in this sample is less than 60. [5]
The times, in months, taken by a builder to build two types of house, $P$ and $Q$, are represented by the independent variables $T_1 \sim N(2.2, 0.4^2)$ and $T_2 \sim N(2.8, 0.5^2)$ respectively.

(i) Find the probability that the total time taken to build one house of each type is less than 6 months.
(ii) Find the probability that the time taken to build a type Q house is more than 1.2 times the time taken to build a type P house. [5]
The random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x} & 2 \leq x \leq 6, \\ 0 & \text{otherwise}, \end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{1}{\ln 3}$. [2]

(ii) Show that $E(X) = 3.64$, correct to 3 significant figures. [3]
(iii) Given that the median of $X$ is $m$, find $P(m < X < E(X))$. [4]
A mill owner claims that the mean mass of sacks of flour produced at his mill is 51 kg. A quality control officer suspects that the mean mass is actually less than 51 kg. In order to test the owner's claim she finds the mass, \( x \) kg, of each of a random sample of 150 sacks and her results are summarised as follows.

\[
\begin{align*}
    n & = 150 \\
    \sum x & = 7480 \\
    \sum x^2 & = 380000
\end{align*}
\]

(i) Carry out the test at the 2.5% significance level. [7]
You may now assume that the population standard deviation of the masses of sacks of flour is 6.856 kg. The quality control officer weighs another random sample of 150 sacks and carries out another test at the 2.5% significance level.

(ii) Given that the population mean mass is 49 kg, find the probability of a Type II error. [5]
If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.