A hollow cylinder with a rough inner surface has radius 0.5 m. A particle $P$ of mass 0.4 kg is in contact with the inner surface of the cylinder. The particle and cylinder rotate together with angular speed $6 \text{ rad s}^{-1}$ about the vertical axis of the cylinder, so that the particle moves in a horizontal circle (see diagram). Given that $P$ is about to slip downwards, find the coefficient of friction between $P$ and the surface of the cylinder. [4]
A small ball is projected from a point 1.5 m above horizontal ground. At a point 9 m above the ground the ball is travelling at 45° above the horizontal and its velocity is 4 m s$^{-1}$. Find the angle of projection of the ball.
One end of a light inextensible string of length 0.4 m is attached to a fixed point $A$ which is above a smooth horizontal surface. A particle $P$ of mass 0.6 kg is attached to the other end of the string. $P$ moves in a circle on the surface with constant speed $v \text{ m s}^{-1}$, with the string taut and making an angle of 60° with the horizontal (see diagram).

(i) Given that $v = 0.5$, calculate the magnitude of the force that the surface exerts on $P$. [4]
(ii) Find the greatest possible value of \( v \) for which \( P \) remains in contact with the surface. [3]
A particle $P$ is projected with speed $25 \text{ m s}^{-1}$ at an angle of $30^\circ$ above the horizontal from a point $O$ on horizontal ground. At time $t$ s after projection the horizontal and vertically upwards displacements of $P$ from $O$ are $x$ m and $y$ m respectively.

(i) Express $x$ and $y$ in terms of $t$ and hence show that the equation of the trajectory of $P$ is

$$y = \frac{x}{\sqrt{3}} - \frac{4x^2}{375}.$$ [4]
(ii) Find the horizontal distance between the two points at which \( P \) is 5 m above the ground. [3]
One end of a light elastic string of natural length 0.8 m and modulus of elasticity 24 N is attached to a fixed point $O$. The other end of the string is attached to a particle $P$ of mass 0.3 kg. $P$ is projected vertically upwards with speed 4 m s$^{-1}$ from a position 1.2 m vertically below $O$.

(i) Calculate the speed of the particle at the position where it is moving with zero acceleration. [5]
(ii) Show that the particle moves 1.2 m while moving upwards with constant deceleration. [3]
A solid object consists of a uniform hemisphere of radius 0.4 m attached to a uniform cylinder of radius 0.4 m so that the circumferences of their circular faces coincide. The hemisphere and cylinder each have weight 20 N. The centre of mass of the object lies at the centre $O$ of their common circular face.

(i) Show that the height of the cylinder is 0.3 m. [2]

A new object is made by cutting the cylinder in half and removing the half not attached to the hemisphere. The cut is perpendicular to the axis of symmetry, so the new object consists of a hemisphere and a cylinder half the height of the original cylinder.

(ii) Find the distance of the centre of mass of the new object from $O$. [4]
The new object is placed with its hemispherical part on a rough horizontal surface. The new object is held in equilibrium by a force of magnitude $P\,\text{N}$ acting along its axis of symmetry, which is inclined at 30° to the horizontal.

(iii) Find $P$. [3]
A particle $P$ of mass $0.2 \text{ kg}$ is released from rest at a point $O$ on a rough plane inclined at $60^\circ$ to the horizontal, and travels down a line of greatest slope. The coefficient of friction between $P$ and the plane is $0.3$. A force of magnitude $0.6x \text{ N}$ acts on $P$ in the direction $PO$, where $x \text{ m}$ is the displacement of $P$ from $O$.

(i) Show that $\frac{dv}{dx} = 5\sqrt{3} - 1.5 - 3x$, where $v \text{ m s}^{-1}$ is the velocity of $P$ at a displacement $x \text{ m}$ from $O$. [3]

(ii) Find the value of $x$ for which $P$ reaches its maximum velocity, and calculate this maximum velocity. [4]
(iii) Calculate the magnitude of the acceleration of $P$ immediately after it has first come to instantaneous rest. [4]