This document consists of 3 printed pages, 1 blank page and 1 insert.
1 (i) It is given that \( x \) satisfies the equation \( 3^{2x} = 5(3^x) + 14 \). Find the value of \( 3^x \) and, using logarithms, find the value of \( x \) correct to 3 significant figures. [4]

(ii) Hence state the values of \( x \) satisfying the equation \( 3^{2|x|} = 5(3^{|x|}) + 14 \). [1]

\[ \text{ln } y \]
\[ (10, 4.77) \]
\[ (5, 3.17) \]

The variables \( x \) and \( y \) satisfy the equation \( y = Ae^{px} \), where \( A \) and \( p \) are constants. The graph of \( \text{ln } y \) against \( x \) is a straight line passing through the points \((5, 3.17)\) and \((10, 4.77)\), as shown in the diagram. Find the values of \( A \) and \( p \) correct to 2 decimal places. [5]

3 A curve has equation \( y = 2 \sin 2x - 5 \cos 2x + 6 \) and is defined for \( 0 \leq x \leq \pi \). Find the \( x \)-coordinates of the stationary points of the curve, giving your answers correct to 3 significant figures. [6]

4 It is given that the positive constant \( a \) is such that

\[ \int_{-a}^{a} (4e^{2x} + 5) \, dx = 100. \]

(i) Show that \( a = \frac{1}{2} \ln(50 + e^{-2a} - 5a) \). [4]

(ii) Use the iterative formula \( a_{n+1} = \frac{1}{2} \ln(50 + e^{-2a_n} - 5a_n) \) to find \( a \) correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

5 (i) Show that \( \frac{\cos 2x + 9 \cos x + 5}{\cos x + 4} \equiv 2 \cos x + 1 \). [3]

(ii) Hence find the exact value of \( \int_{-\pi}^{\pi} \frac{\cos 4x + 9 \cos 2x + 5}{\cos 2x + 4} \, dx \). [4]

6 The equation of a curve is \( 3x^2 + 4xy + y^2 = 24 \). Find the equation of the normal to the curve at the point \((1, 3)\), giving your answer in the form \( ax + by + c = 0 \) where \( a \), \( b \) and \( c \) are integers. [8]
7 The polynomial \( p(x) \) is defined by

\[
p(x) = ax^3 + 3x^2 + bx + 12,
\]

where \( a \) and \( b \) are constants. It is given that \( (x + 3) \) is a factor of \( p(x) \). It is also given that the remainder is 18 when \( p(x) \) is divided by \( (x + 2) \).

(i) Find the values of \( a \) and \( b \). \[5\]

(ii) When \( a \) and \( b \) have these values,

(a) show that the equation \( p(x) = 0 \) has exactly one real root, \[4\]

(b) solve the equation \( p(\sec y) = 0 \) for \(-180^\circ < y < 180^\circ\). \[3\]