1. Use the trapezium rule with four intervals to find an approximation to
\[ \int_{1}^{5} (2^x - 8) \, dx. \] [3]

2. The variables \( x \) and \( y \) satisfy the equation \( y = a(b^x) \), where \( a \) and \( b \) are constants. The graph of \( \ln y \) against \( x \) is a straight line passing through the points (0.75, 1.70) and (1.53, 2.18), as shown in the diagram. Find the values of \( a \) and \( b \) correct to 2 decimal places. [5]

3. (a) Find \( \int 4 \cos^2\left(\frac{1}{2}\theta\right) \, d\theta \). [3]

(b) Find the exact value of \( \int_{-1}^{6} \frac{1}{2x + 3} \, dx \). [4]

4. For each of the following curves, find the exact gradient at the point indicated:

(i) \( y = 3 \cos 2x - 5 \sin x \) at \( \left(\frac{1}{6}\pi, -1\right) \). [3]

(ii) \( x^3 + 6xy + y^3 = 21 \) at \((1, 2)\). [5]
5 (i) Given that \((x + 2)\) and \((x + 3)\) are factors of
\[5x^3 + ax^2 + b,\]
find the values of the constants \(a\) and \(b\). [4]

(ii) When \(a\) and \(b\) have these values, factorise
\[5x^3 + ax^2 + b\]
completely, and hence solve the equation
\[5^{3y+1} + a \times 5^y + b = 0,\]
giving any answers correct to 3 significant figures. [5]

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The diagram shows part of the curve \(y = \frac{x^2}{1 + e^{3x}}\) and its maximum point \(M\). The \(x\)-coordinate of \(M\) is denoted by \(m\).

(i) Find \(\frac{dy}{dx}\) and hence show that \(m\) satisfies the equation \(x = \frac{2}{5}(1 + e^{-3x})\). [4]

(ii) Show by calculation that \(m\) lies between 0.7 and 0.8. [2]

(iii) Use an iterative formula based on the equation in part (i) to find \(m\) correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

7 The angle \(\alpha\) lies between 0° and 90° and is such that
\[2 \tan^2 \alpha + \sec^2 \alpha = 5 - 4 \tan \alpha.\]

(i) Show that
\[3 \tan^2 \alpha + 4 \tan \alpha - 4 = 0\]
and hence find the exact value of \(\tan \alpha\). [4]

(ii) It is given that the angle \(\beta\) is such that \(\cot(\alpha + \beta) = 6\). Without using a calculator, find the exact value of \(\cot \beta\). [5]