READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in. Write in dark blue or black pen. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an electronic calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 50. Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The diagram shows the graph of the probability density function, $f$, of a random variable $X$. Find the median of $X$. [3]

2 The heights of a certain type of plant have a normal distribution. When the plants are grown without fertilizer, the population mean and standard deviation are 24.0 cm and 4.8 cm respectively. A gardener wishes to test, at the 2% significance level, whether Hiergro fertilizer will increase the mean height. He treats 150 randomly chosen plants with Hiergro and finds that their mean height is 25.0 cm. Assuming that the standard deviation of the heights of plants treated with Hiergro is still 4.8 cm, carry out the test. [5]

3 The cost of hiring a bicycle consists of a fixed charge of 500 cents together with a charge of 3 cents per minute. The number of minutes for which people hire a bicycle has mean 142 and standard deviation 35.

(i) Find the mean and standard deviation of the amount people pay when hiring a bicycle. [3]

(ii) 6 people hire bicycles independently. Find the mean and standard deviation of the total amount paid by all 6 people. [3]

4 A cereal manufacturer claims that 25% of cereal packets contain a free gift. Lola suspects that the true proportion is less than 25%. In order to test the manufacturer’s claim at the 5% significance level, she checks a random sample of 20 packets.

(i) Find the critical region for the test. [5]

(ii) Hence find the probability of a Type I error. [1]

Lola finds that 2 packets in her sample contain a free gift.

(iii) State, with a reason, the conclusion she should draw. [2]
A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x-1} & 3 \leq x \leq 5, \\ 0 & \text{otherwise}, \end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{1}{\ln 2}$. [4]

(ii) Find $a$ such that $P(X < a) = 0.75$. [4]

In order to obtain a random sample of people who live in her town, Jane chooses people at random from the telephone directory for her town.

(i) Give a reason why Jane’s method will not give a random sample of people who live in the town. [1]

Jane now uses a valid method to choose a random sample of 200 people from her town and finds that 38 live in apartments.

(ii) Calculate an approximate 99% confidence interval for the proportion of all people in Jane’s town who live in apartments. [4]

(iii) Jane uses the same sample to give a confidence interval of width 0.1 for this proportion. This interval is an $x\%$ confidence interval. Find the value of $x$. [4]

A random variable $X$ has the distribution Po$(1.6)$.

(i) The random variable $R$ is the sum of three independent values of $X$. Find $P(R < 4)$. [3]

(ii) The random variable $S$ is the sum of $n$ independent values of $X$. It is given that

$$P(S = 4) = \frac{16}{3} \times P(S = 2).$$

Find $n$. [4]

(iii) The random variable $T$ is the sum of 40 independent values of $X$. Find $P(T > 75)$. [4]