1 Solve the inequality \(|x + 2| > \frac{1}{2}x - 2\). [4]

2 Use logarithms to solve the equation \(4^{x+1} = 5^{2x-3}\), giving your answer correct to 3 significant figures. [4]

3 The diagram shows the curve \(y = x - 2 \ln x\) and its minimum point \(M\).

(i) Find the \(x\)-coordinate of \(M\). [2]

(ii) Use the trapezium rule with three intervals to estimate the value of

\[ \int_{\frac{5}{2}}^{5} (x - 2 \ln x) \, dx, \]

giving your answer correct to 2 decimal places. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (ii). [1]

4 Find the exact value of the positive constant \(k\) for which

\[ \int_{0}^{k} e^{4x} \, dx = \int_{0}^{2k} e^{x} \, dx. \] [6]

5 (i) By sketching a suitable pair of graphs, show that the equation

\[ \frac{1}{x} = \sin x, \]

where \(x\) is in radians, has only one root for \(0 < x \leq \frac{1}{2} \pi\). [2]

(ii) Verify by calculation that this root lies between \(x = 1.1\) and \(x = 1.2\). [2]

(iii) Use the iterative formula \(x_{n+1} = \frac{1}{\sin x_n}\) to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]
The parametric equations of a curve are
\[ x = 1 + 2 \sin^2 \theta, \quad y = 4 \tan \theta. \]

(i) Show that \( \frac{dy}{dx} = \frac{1}{\sin \theta \cos^3 \theta} \). [3]

(ii) Find the equation of the tangent to the curve at the point where \( \theta = \frac{1}{4} \pi \), giving your answer in the form \( y = mx + c \). [4]

The polynomial \( ax^3 - 3x^2 - 11x + b \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (x + 2) \) is a factor of \( p(x) \), and that when \( p(x) \) is divided by \( (x + 1) \) the remainder is 12.

(i) Find the values of \( a \) and \( b \). [5]

(ii) When \( a \) and \( b \) have these values, factorise \( p(x) \) completely. [3]

Express \( 5 \cos \theta - 3 \sin \theta \) in the form \( R \cos(\theta + \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), giving the exact value of \( R \) and the value of \( \alpha \) correct to 2 decimal places. [3]

(ii) Hence solve the equation
\[ 5 \cos \theta - 3 \sin \theta = 4, \]
giving all solutions in the interval \( 0^\circ \leq \theta \leq 360^\circ \). [4]

(iii) Write down the least value of \( 15 \cos \theta - 9 \sin \theta \) as \( \theta \) varies. [1]