READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
1. Solve the inequality $|3x + 1| > 8$. [3]

2. The sequence of values given by the iterative formula

$$x_{n+1} = \frac{7x_n}{8} + \frac{5}{2x_n^2},$$

with initial value $x_1 = 1.7$, converges to $\alpha$.

(i) Use this iterative formula to determine $\alpha$ correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation that is satisfied by $\alpha$ and hence show that $\alpha = \sqrt{20}$. [2]

3. The polynomial $x^3 + 4x^2 + ax + 2$, where $a$ is a constant, is denoted by $p(x)$. It is given that the remainder when $p(x)$ is divided by $(x + 1)$ is equal to the remainder when $p(x)$ is divided by $(x - 2)$.

(i) Find the value of $a$. [3]

(ii) When $a$ has this value, show that $(x - 1)$ is a factor of $p(x)$ and find the quotient when $p(x)$ is divided by $(x - 1)$. [3]

4. (a) Find $\int e^{1-2x} \, dx$. [2]

(b) Express $\sin^2 3x$ in terms of $\cos 6x$ and hence find $\int \sin^2 3x \, dx$. [4]

5. The variables $x$ and $y$ satisfy the equation $y = A(b^x)$, where $A$ and $b$ are constants. The graph of $\ln y$ against $x$ is a straight line passing through the points $(1.4, 0.8)$ and $(2.2, 1.2)$, as shown in the diagram. Find the values of $A$ and $b$, correct to 2 decimal places. [6]
6  (i) Express \(2 \sin \theta - \cos \theta\) in the form \(R \sin(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), giving the exact value of \(R\) and the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence solve the equation
\[
2 \sin \theta - \cos \theta = -0.4,
\]
giving all solutions in the interval \(0^\circ \leq \theta \leq 360^\circ\). [4]

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The diagram shows the curve \(y = \frac{\ln x}{x^2}\) and its maximum point \(M\).

(i) Find the exact coordinates of \(M\). [5]

(ii) Use the trapezium rule with three intervals to estimate the value of
\[
\int_1^4 \frac{\ln x}{x^2} \, dx,
\]
giving your answer correct to 2 decimal places. [3]

8  The equation of a curve is
\[
x^2 + 2xy - y^2 + 8 = 0.
\]

(i) Show that the tangent to the curve at the point \((-2, 2)\) is parallel to the \(x\)-axis. [4]

(ii) Find the equation of the tangent to the curve at the other point on the curve for which \(x = -2\), giving your answer in the form \(y = mx + c\). [5]