This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2010 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep”) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

**AEF**  Any Equivalent Form (of answer is equally acceptable)

**AG**  Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

**BOD**  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

**CAO**  Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

**CWO**  Correct Working Only - often written by a ‘fortuitous' answer

**ISW**  Ignore Subsequent Working

**MR**  Misread

**PA**  Premature Approximation (resulting in basically correct work that is insufficiently accurate)

**SOS**  See Other Solution (the candidate makes a better attempt at the same question)

**SR**  Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

**MR -1**  A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

**PA -1**  This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.
1 EITHER: State or imply non-modular inequality \((2(x - 3))^2 > (3x + 1)^2\), or corresponding quadratic equation, or pair of linear equations \(2(x - 3) = \pm(3x + 1)\) B1
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
Obtain critical values \(x = -7\) and \(x = 1\) A1
State answer \(-7 < x < 1\) A1
OR: Obtain critical value \(x = -7\) or \(x = 1\) from a graphical method, or by inspection, or by solving a linear equation or inequality B1
Obtain critical values \(x = -7\) and \(x = 1\) B2
State answer \(-7 < x < 1\) B1 [4]

[Do not condone: < for <.]

2 Use law for the logarithm of a power, a quotient, or a product correctly at least once M1
Use \(\ln e = 1\) or \(e = \exp(1)\) M1
Obtain a correct equation free of logarithms, e.g. \(1 + x^2 = e^x\) A1
Solve and obtain answer \(x = 0.763\) only A1 [4]
[For the solution \(x = 0.763\) with no relevant working give B1, and a further B1 if 0.763 is shown to be the only root.]
[Treat the use of logarithms to base 10 with answer 0.333 only, as a misread.]
[SR: Allow iteration, giving B1 for an appropriate formula, e.g. \(x_{n+1} = \exp((\ln(1 + x_n^2) - 1)/2)\), M1 for using it correctly once, A1 for 0.763, and A1 for showing the equation has no other root but 0.763.]  

3 Attempt use of \(\cos(A + B)\) formula to obtain an equation in \(\cos \theta\) and \(\sin \theta\) M1
Use trig formula to obtain an equation in \(\tan \theta\) (or \(\cos \theta\), \(\sin \theta\) or \(\cot \theta\)) M1
Obtain \(\tan \theta = 1/(4 + \sqrt{3})\) or equivalent (or find \(\cos \theta\), \(\sin \theta\) or \(\cot \theta\)) A1
Obtain \(\theta = 9.9^\circ\) A1
Obtain \(\theta = 189.9^\circ\), and no others in the given interval A1 [5]
[Ignore answers outside the given interval. Treat answers in radians as a misread (0.173, 3.31).]

[The other solution methods are via \(\cos \theta = \pm(4 + \sqrt{3})/\sqrt{1+\left(4+\sqrt{3}\right)^2}\) or \(\sin \theta = \pm1/\sqrt{1+\left(4+\sqrt{3}\right)^2}\).]

4 (i) Make recognisable sketch of a relevant graph over the given range B1
Sketch the other relevant graph on the same diagram and justify the given statement B1 [2]

(ii) Consider sign of \(4x^2 - 1 - \cot x\) at \(x = 0.6\) and \(x = 1\), or equivalent M1
Complete the argument correctly with correct calculated values A1 [2]

(iii) Use the iterative formula correctly at least once M1
Obtain final answer 0.73 A1
Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval \((0.725, 0.735)\) A1 [3]
5 (i) State or imply \( dx = 2 \cos \theta \, d\theta \), or \( \frac{dx}{d\theta} = 2 \cos \theta \), or equivalent \( \quad B1 \)

Substitute for \( x \) and \( dx \) throughout the integral \( \quad M1 \)

Obtain the given answer correctly, having changed limits and shown sufficient working \( \quad A1 \) \[3\]

(ii) Replace integrand by \( 2 - 2 \cos 2\theta \), or equivalent \( \quad B1 \)

Obtain integral \( 2 \theta - \sin 2\theta \), or equivalent \( \quad B1 \)

Substitute limits correctly in an integral of the form \( a\theta \pm b \sin 2\theta \), where \( ab \gtrless 0 \) \( \quad M1 \)

Obtain answer \( \frac{1}{3}\pi - \frac{\sqrt{3}}{2} \) or exact equivalent \( \quad A1 \) \[4\]

[The f.t. is on integrands of the form \( a + c \cos 2\theta \), where \( ac \gtrless 0 \).]

6 (i) State modulus is 2 \( \quad B1 \)

State argument is \( \frac{1}{6}\pi \), or \( 30^\circ \), or 0.524 radians \( \quad B1 \) \[2\]

(ii) (a) State answer \( 3\sqrt{3} + i \) \( \quad B1 \)

(b) \( EITHER: \) Multiply numerator and denominator by \( \sqrt{3} - i \), or equivalent \( \quad M1 \)

Simplify denominator to 4 or numerator to \( 2\sqrt{3} + 2i \) \( \quad A1 \)

Obtain final answer \( \frac{1}{2}\sqrt{3} + \frac{1}{2}i \), or equivalent \( \quad A1 \)

\( OR 1: \) Obtain two equations in \( x \) and \( y \) and solve for \( x \) or for \( y \) \( \quad M1 \)

Obtain \( x = \frac{1}{2}\sqrt{3} \) or \( y = \frac{1}{2} \) \( \quad A1 \)

Obtain final answer \( \frac{1}{2}\sqrt{3} + \frac{1}{2}i \), or equivalent \( \quad A1 \)

\( OR 2: \) Using the correct processes express \( iz/\bar{z} \) in polar form \( \quad M1 \)

Obtain \( x = \frac{1}{2}\sqrt{3} \) or \( y = \frac{1}{2} \) \( \quad A1 \)

Obtain final answer \( \frac{1}{2}\sqrt{3} + \frac{1}{2}i \), or equivalent \( \quad A1 \) \[4\]

(iii) Plot \( A \) and \( B \) in relatively correct positions \( \quad B1 \)

\( EITHER: \) Use fact that angle \( AOB = \arg(iz*) - \arg z \) \( \quad M1 \)

Obtain the given answer \( \quad A1 \)

\( OR 1: \) Obtain \( \tan AOB \) from gradients of \( OA \) and \( OB \) and the correct \( \tan(A - B) \) formula \( \quad M1 \)

Obtain the given answer \( \quad A1 \)

\( OR 2: \) Obtain \( \cos AOB \) by using correct cosine formula or scalar product \( \quad M1 \)

Obtain the given answer \( \quad A1 \) \[3\]
7 (i) State correct equation in any form, e.g., \( \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \) \( \text{B1} \) [1]

(ii) \textit{EITHER:} Equate a relevant scalar product to zero and form an equation in \( \lambda \) \( \text{M1} \)

\textit{OR 1:} Equate derivative of \( \mathbf{OP}^2 \) (or \( \mathbf{OP} \)) to zero and form an equation in \( \lambda \) \( \text{M1} \)

\textit{OR 2:} Use Pythagoras in \( \mathbf{OAP} \) or \( \mathbf{OBP} \) and form an equation in \( \lambda \) \( \text{M1} \)

State a correct equation in any form \( \text{A1} \)

Solve and obtain \( \lambda = -\frac{1}{6} \) or equivalent \( \text{A1} \)

Obtain final answer \( \overrightarrow{OP} = \frac{2}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} + \frac{7}{3}\mathbf{k} \), or equivalent \( \text{A1} \) [4]

(iii) \textit{EITHER:} State or imply \( \overrightarrow{OP} \) is a normal to the required plane \( \text{M1} \)

State normal vector \( 2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \), or equivalent \( \text{A1} \)√

Substitute coordinates of a relevant point in \( 2x + 5y + 7z = d \) and evaluate \( d \) \( \text{M1} \)

Obtain answer \( 2x + 5y + 7z = 26 \), or equivalent \( \text{A1} \)

\textit{OR 1:} Find a vector normal to plane \( \mathbf{AOB} \) and calculate its vector product with a direction vector for the line \( \mathbf{AB} \) \( \text{M1}^* \)

Obtain answer \( 2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \), or equivalent \( \text{A1} \)

Substitute coordinates of a relevant point in \( 2x + 5y + 7z = d \) and evaluate \( d \) \( \text{M1(dep*)} \)

Obtain answer \( 2x + 5y + 7z = 26 \), or equivalent \( \text{A1} \)

\textit{OR 2:} Set up and solve simultaneous equations in \( a, b, c \) derived from zero scalar products of \( a\mathbf{i} + b\mathbf{j} + c\mathbf{k} \) with (i) a direction vector for line \( \mathbf{AB} \), (ii) a normal to plane \( \mathbf{OAB} \) \( \text{M1}^* \)

Obtain \( a : b : c = 2 : 5 : 7 \), or equivalent \( \text{A1} \)

Substitute coordinates of a relevant point in \( 2x + 5y + 7z = d \) and evaluate \( d \) \( \text{M1(dep*)} \)

Obtain answer \( 2x + 5y + 7z = 26 \), or equivalent \( \text{A1} \)

\textit{OR 3:} With \( Q(x, y, z) \) on plane, use Pythagoras in \( \mathbf{OPQ} \) to form an equation in \( x, y \) and \( z \) \( \text{M1}^* \)

Form a correct equation \( \text{A1} \)√

Reduce to linear form \( \text{M1(dep*)} \)

Obtain answer \( 2x + 5y + 7z = 26 \), or equivalent \( \text{A1} \)

\textit{OR 4:} Find a vector normal to plane \( \mathbf{AOB} \) and form a 2-parameter equation with relevant vectors, e.g., \( \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + \mu(8\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \) \( \text{M1}^* \)

State three correct equations in \( x, y, z, \lambda \) and \( \mu \) \( \text{A1} \)

Eliminate \( \lambda \) and \( \mu \) \( \text{M1(dep*)} \)

Obtain answer \( 2x + 5y + 7z = 26 \), or equivalent \( \text{A1} \) [4]
8 (i) State or imply the form \( \frac{A}{1+x} + \frac{Bx+C}{1+2x^2} \) \( B1 \)

Use any relevant method to evaluate a constant \( M1 \)
Obtain one of \( A = -1, B = 2, C = 1 \) \( A1 \)
Obtain a second value \( A1 \)
Obtain the third value \( A1 \) \[5\]

(ii) Use correct method to obtain the first two terms of the expansion of \((1+x)^{-1}\) or \((1+2x^2)^{-1}\) \( M1 \)
Obtain correct expansion of each partial fraction as far as necessary \( A1\sqrt{+} A1\sqrt{+} \)
Multiply out fully by \( Bx + C \), where \( BC \neq 0 \) \( M1 \)
Obtain answer \( 3x - 3x^2 - 3x^3 \) \( A1 \) \[5\]

(Symbolic binomial coefficients, e.g., \( \left( \begin{array}{cc} -1 \\ 1 \end{array} \right) \) are not sufficient for the first \( M1 \). The f.t. is on \( A, B, C \).)

[If \( B \) or \( C \) omitted from the form of fractions, give \( B0M1A0A0A0 \) in (i); \( M1A1\sqrt{+}A1\sqrt{+} \) in (ii), max 4/10.]

[If a constant \( D \) is added to the correct form, give \( M1A1A1A1 \) and \( B1 \) if and only if \( D = 0 \) is stated.]

[If an extra term \( D/(1+2x^2) \) is added, give \( B1M1A1A1 \), and \( A1 \) if \( C + D = 1 \) is resolved to \( 1/(1+2x^2) \).]

[In the case of an attempt to expand \( 3x(1+x)^{-1}(1+2x^2)^{-1} \), give \( M1A1A1 \) for the expansions up to the term in \( x^2 \), \( M1 \) for multiplying out fully, and \( A1 \) for the final answer.]

[For the identity \( 3x = (1+x+2x^2+2x^3)(a+bx+cx^2+dx^3) \) give \( M1A1 \); then \( M1A1 \) for using a relevant method to find two of \( a = 0, b = 3, c = -3 \) and \( d = -3 \); and then \( A1 \) for the final answer in series form.]

9 (i) Use correct product rule \( M1 \)
Obtain correct derivative in any form \( A1 \)
Equate derivative to zero and find non-zero \( x \) \( M1 \)
Obtain \( x = \exp\left(-\frac{1}{4}\right) \), or equivalent \( A1 \)
Obtain \( y = -1/(3e) \), or any ln-free equivalent \( A1 \) \[5\]

(ii) Integrate and reach \( kx^4 \ln x + l \int x^4 \frac{1}{x} \text{dx} \) \( M1 \)
Obtain \( \frac{1}{4}x^4 \ln x - \frac{1}{4} \int x^3 \text{dx} \) \( A1 \)
Obtain integral \( \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 \), or equivalent \( A1 \)
Use limits \( x = 1 \) and \( x = 2 \) correctly, having integrated twice \( M1 \)
Obtain answer \( 4\ln 2 - \frac{15}{16} \), or exact equivalent \( A1 \) \[5\]
10 (i) State or imply \( \frac{dx}{dt} = k(20 - x) \)

Show that \( k = 0.05 \) B1 [2]

(ii) Separate variables correctly and integrate both sides B1

Obtain term \(-\ln(20 - x)\), or equivalent B1

Obtain term \( \frac{1}{20} t \), or equivalent B1

Evaluate a constant or use limits \( t = 0, x = 0 \) in a solution containing terms \( a \ln(20 - x) \) and \( bt \) M1*

Obtain correct answer in any form, e.g. \( \ln 20 - \ln(20 - x) = \frac{1}{20} t \) A1 [5]

(iii) Substitute \( t = 10 \) and calculate \( x \) M1(dep*)

Obtain answer \( x = 7.9 \) A1 [2]

(iv) State that \( x \) approaches 20 B1 [1]