1 2% of biscuits on a production line are broken. Broken biscuits occur randomly. 180 biscuits are checked to see whether they are broken. Use a suitable approximation to find the probability that fewer than 4 are broken. [3]

2 The lengths of sewing needles in travel sewing kits are distributed normally with mean \( \mu \) mm and standard deviation 1.5 mm. A random sample of \( n \) needles is taken. Find the smallest value of \( n \) such that the width of a 95% confidence interval for the population mean is at most 1 mm. [4]

3 The weights of pebbles on a beach are normally distributed with mean 48.5 grams and standard deviation 12.4 grams.

(i) Find the probability that the mean weight of a random sample of 5 pebbles is greater than 51 grams. [3]

(ii) The probability that the mean weight of a random sample of \( n \) pebbles is less than 51.6 grams is 0.9332. Find the value of \( n \). [4]

4 The number of severe floods per year in a certain country over the last 100 years has followed a Poisson distribution with mean 1.8. Scientists suspect that global warming has now increased the mean. A hypothesis test, at the 5% significance level, is to be carried out to test this suspicion. The number of severe floods, \( X \), that occur next year will be used for the test.

(i) Show that the rejection region for the test is \( X > 4 \). [5]

(ii) Find the probability of making a Type II error if the mean number of severe floods is now actually 2.3. [3]

5 The continuous random variable \( X \) has probability density function given by

\[
f(x) = \begin{cases} 
  k \cos x & 0 \leq x \leq \frac{\pi}{4}, \\
  0 & \text{otherwise}, 
\end{cases}
\]

where \( k \) is a constant.

(i) Show that \( k = \sqrt{2} \). [2]

(ii) Find \( P(X > 0.4) \). [2]

(iii) Find the upper quartile of \( X \). [3]

(iv) Find the probability that exactly 3 out of 5 random observations of \( X \) have values greater than the upper quartile. [2]
Photographers often need to take many photographs of families until they find a photograph which everyone in the family likes. The number of photographs taken until obtaining one which everybody likes has mean 15.2. A new photographer claims that she can obtain a photograph which everybody likes with fewer photographs taken. To test at the 10% level of significance whether this claim is justified, the numbers of photographs, $x$, taken by the new photographer with a random sample of 60 families are recorded. The results are summarised by $\Sigma x = 890$ and $\Sigma x^2 = 13780$.

(i) Calculate unbiased estimates of the population mean and variance of the number of photographs taken by the new photographer. [3]

(ii) State null and alternative hypotheses for the test, and state also the probability that the test results in a Type I error. Say what a Type I error means in the context of the question. [3]

(iii) Carry out the test. [4]

The volume of liquid in cans of cola is normally distributed with mean 330 millilitres and standard deviation 5.2 millilitres. The volume of liquid in bottles of tonic water is normally distributed with mean 500 millilitres and standard deviation 7.1 millilitres.

(i) Find the probability that 3 randomly chosen cans of cola contain less liquid than 2 randomly chosen bottles of tonic water. [5]

(ii) A new drink is made by mixing the contents of 2 cans of cola with half a bottle of tonic water. Find the probability that the volume of the new drink is more than 900 millilitres. [4]