UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS 9709/07
Paper 7 Probability & Statistics 2 (S2) October/November 2008
1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in
degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger
numbers of marks later in the paper.
1. Alan wishes to choose one child at random from the eleven children in his music class. The children are numbered 2, 3, 4, and so on, up to 12. Alan then throws two fair dice, each numbered from 1 to 6, and chooses the child whose number is the sum of the scores on the two dice.

(i) Explain why this is an unsatisfactory method of choosing a child. [2]

(ii) Describe briefly a satisfactory method of choosing a child. [2]

2. The times taken for the pupils in Ming’s year group to do their English homework have a normal distribution with standard deviation 15.7 minutes. A teacher estimates that the mean time is 42 minutes. The times taken by a random sample of 3 students from the year group were 27, 35 and 43 minutes. Carry out a hypothesis test at the 10% significance level to determine whether the teacher’s estimate for the mean should be accepted, stating the null and alternative hypotheses. [5]

3. Weights of garden tables are normally distributed with mean 36 kg and standard deviation 1.6 kg. Weights of garden chairs are normally distributed with mean 7.3 kg and standard deviation 0.4 kg. Find the probability that the total weight of 2 randomly chosen tables is more than the total weight of 10 randomly chosen chairs. [5]

4. Diameters of golf balls are known to be normally distributed with mean $\mu$ cm and standard deviation $\sigma$ cm. A random sample of 130 golf balls was taken and the diameters, $x$ cm, were measured. The results are summarised by $\sum x = 555.1$ and $\sum x^2 = 2371.30$.

(i) Calculate unbiased estimates of $\mu$ and $\sigma^2$. [3]

(ii) Calculate a 97% confidence interval for $\mu$. [3]

(iii) 300 random samples of 130 balls are taken and a 97% confidence interval is calculated for each sample. How many of these intervals would you expect not to contain $\mu$? [1]

5. Every month Susan enters a particular lottery. The lottery company states that the probability, $p$, of winning a prize is 0.0017 each month. Susan thinks that the probability of winning is higher than this, and carries out a test based on her 12 lottery results in a one-year period. She accepts the null hypothesis $p = 0.0017$ if she has no wins in the year and accepts the alternative hypothesis $p > 0.0017$ if she wins a prize in at least one of the 12 months.

(i) Find the probability of the test resulting in a Type I error. [2]

(ii) If in fact the probability of winning a prize each month is 0.0024, find the probability of the test resulting in a Type II error. [3]

(iii) Use a suitable approximation, with $p = 0.0024$, to find the probability that in a period of 10 years Susan wins a prize exactly twice. [3]
In their football matches, Rovers score goals independently and at random times. Their average rate of scoring is 2.3 goals per match.

(i) State the expected number of goals that Rovers will score in the first half of a match. [1]

(ii) Find the probability that Rovers will not score any goals in the first half of a match but will score one or more goals in the second half of the match. [2]

(iii) Football matches last for 90 minutes. In a particular match, Rovers score one goal in the first 30 minutes. Find the probability that they will score at least one further goal in the remaining 60 minutes. [3]

Independently of the number of goals scored by Rovers, the number of goals scored per football match by United has a Poisson distribution with mean 1.8.

(iv) Find the probability that a total of at least 3 goals will be scored in a particular match when Rovers play United. [3]

The time in hours taken for clothes to dry can be modelled by the continuous random variable with probability density function given by

\[ f(t) = \begin{cases} 
  k \sqrt{t} & 1 \leq t \leq 4, \\
  0 & \text{otherwise,}
\end{cases} \]

where \( k \) is a constant.

(i) Show that \( k = \frac{3}{14} \). [3]

(ii) Find the mean time taken for clothes to dry. [4]

(iii) Find the median time taken for clothes to dry. [3]

(iv) Find the probability that the time taken for clothes to dry is between the mean time and the median time. [2]