1 The time taken for Samuel to drive home from work is distributed with mean 46 minutes. Samuel discovers a different route and decides to test at the 5% level whether the mean time has changed. He tries this route on a large number of different days chosen randomly and calculates the mean time.

(i) State the null and alternative hypotheses for this test. [1]

(ii) Samuel calculates the value of his test statistic $z$ to be $-1.729$. What conclusion can he draw? [2]

2 (i) Write down the mean and variance of the distribution of the means of random samples of size $n$ taken from a very large population having mean $\mu$ and variance $\sigma^2$. [2]

(ii) What, if anything, can you say about the distribution of sample means

(a) if $n$ is large, [1]

(b) if $n$ is small? [1]

3 A survey was conducted to find the proportion of people owning DVD players. It was found that 203 out of a random sample of 278 people owned a DVD player.

(i) Calculate a 97% confidence interval for the true proportion of people who own a DVD player. [4]

A second survey to find the proportion of people owning DVD players was conducted at 10 o’clock on a Thursday morning in a shopping centre.

(ii) Give one reason why this is not a satisfactory sample. [1]

4 In summer, wasps’ nests occur randomly in the south of England at an average rate of 3 nests for every 500 houses.

(i) Find the probability that two villages in the south of England, with 600 houses and 700 houses, have a total of exactly 3 wasps’ nests. [3]

(ii) Use a suitable approximation to estimate the probability of there being fewer than 369 wasps’ nests in a town with 64 000 houses. [4]

5 Climbing ropes produced by a manufacturer have breaking strengths which are normally distributed with mean 160 kg and standard deviation 11.3 kg. A group of climbers have weights which are normally distributed with mean 66.3 kg and standard deviation 7.1 kg.

(i) Find the probability that a rope chosen randomly will break under the combined weight of 2 climbers chosen randomly. [5]

Each climber carries, in a rucksack, equipment amounting to half his own weight.

(ii) Find the mean and variance of the combined weight of a climber and his rucksack. [3]

(iii) Find the probability that the combined weight of a climber and his rucksack is greater than 87 kg. [2]
6 Pieces of metal discovered by people using metal detectors are found randomly in fields in a certain area at an average rate of 0.8 pieces per hectare. People using metal detectors in this area have a theory that ploughing the fields increases the average number of pieces of metal found per hectare. After ploughing, they tested this theory and found that a randomly chosen field of area 3 hectares yielded 5 pieces of metal.

(i) Carry out the test at the 10% level of significance. [6]

(ii) What would your conclusion have been if you had tested at the 5% level of significance? [1]

Jack decides that he will reject the null hypothesis that the average number is 0.8 pieces per hectare if he finds 4 or more pieces of metal in another ploughed field of area 3 hectares.

(iii) If the true mean after ploughing is 1.4 pieces per hectare, calculate the probability that Jack makes a Type II error. [3]

7 At a town centre car park the length of stay in hours is denoted by the random variable \( X \), which has probability density function given by

\[
    f(x) = \begin{cases} 
        kx^{-\frac{3}{2}} & 1 \leq x \leq 9, \\
        0 & \text{otherwise}, 
    \end{cases}
\]

where \( k \) is a constant.

(i) Interpret the inequalities \( 1 \leq x \leq 9 \) in the definition of \( f(x) \) in the context of the question. [1]

(ii) Show that \( k = \frac{3}{4} \). [2]

(iii) Calculate the mean length of stay. [3]

The charge for a length of stay of \( x \) hours is \( (1 - e^{-x}) \) dollars.

(iv) Find the length of stay for the charge to be at least 0.75 dollars [3]

(v) Find the probability of the charge being at least 0.75 dollars. [2]