1 Solve the inequality $|2x - 1| > |x|$. [4]

2 (i) Express $4^x$ in terms of $y$, where $y = 2^x$. [1]
(ii) Hence find the values of $x$ that satisfy the equation

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places. [5]

3 The polynomial $4x^3 - 7x + a$, where $a$ is a constant, is denoted by $p(x)$. It is given that $(2x - 3)$ is a factor of $p(x)$.

(i) Show that $a = -3$. [2]
(ii) Hence, or otherwise, solve the equation $p(x) = 0$. [4]

4 (i) Prove the identity

$$\tan(x + 45^\circ) - \tan(45^\circ - x) \equiv 2 \tan 2x.$$ [4]

(ii) Hence solve the equation

$$\tan(x + 45^\circ) - \tan(45^\circ - x) = 2,$$

for $0^\circ \leq x \leq 180^\circ$. [3]

5 The diagram shows a chord joining two points, $A$ and $B$, on the circumference of a circle with centre $O$ and radius $r$. The angle $AOB$ is $\alpha$ radians, where $0 < \alpha < \pi$. The area of the shaded segment is one sixth of the area of the circle.

(i) Show that $\alpha$ satisfies the equation

$$x = \frac{1}{3}\pi + \sin x.$$ [3]

(ii) Verify by calculation that $\alpha$ lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \frac{1}{3}\pi + \sin x_n,$$

with initial value $x_1 = 2$, to determine $\alpha$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]
The diagram shows the part of the curve $y = \frac{e^{2x}}{x}$ for $x > 0$, and its minimum point $M$.

(i) Find the coordinates of $M$. \[5\]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$
\int_1^2 \frac{e^{2x}}{x} \, dx,
$$

giving your answer correct to 1 decimal place. \[3\]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (ii). \[1\]

7 (i) Given that $y = \tan 2x$, find $\frac{dy}{dx}$. \[2\]

(ii) Hence, or otherwise, show that

$$
\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx = \frac{1}{2} \sqrt{3},
$$

and, by using an appropriate trigonometrical identity, find the exact value of $\int_0^{\frac{\pi}{2}} \tan^2 2x \, dx$. \[6\]

(iii) Use the identity $\cos 4x \equiv 2 \cos^2 2x - 1$ to find the exact value of

$$
\int_0^{\frac{\pi}{6}} \frac{1}{1 + \cos 4x} \, dx.
$$

\[2\]