MARK SCHEME for the October/November 2006 question paper

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2006 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Mark Scheme Notes

• Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol \( \sqrt{\text{ }} \) implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking \( g \) equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

- **AEF** Any Equivalent Form (of answer is equally acceptable)
- **AG** Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- **BOD** Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
- **CAO** Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- **CWO** Correct Working Only – often written by a ‘fortuitous’ answer
- **ISW** Ignore Subsequent Working
- **MR** Misread
- **PA** Premature Approximation (resulting in basically correct work that is insufficiently accurate)
- **SOS** See Other Solution (the candidate makes a better attempt at the same question)
- **SR** Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

- **MR -1** A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- **PA -1** This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.
1 \textit{EITHER}: State or imply non-modular inequality \((2x-1)^2 > x^2\) or corresponding quadratic equation
\begin{align*}
or pair of linear equations \quad 2x - 1 = \pm x
\end{align*}
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations
\begin{align*}
\text{Obtain critical values } x = 1 \text{ and } x = \frac{1}{3}
\end{align*}
State answer \(x < \frac{1}{3}, \ x > 1\)
\begin{align*}
\text{OR: } \text{Obtain critical value } x = 1 \text{ from a graphical method, or by inspection, or by solving a linear inequality or linear equation}
\end{align*}
\begin{align*}
\text{Obtain the critical value } x = \frac{1}{3} \text{ similarly}
\end{align*}
State answer \(x < \frac{1}{3}, \ x > 1\)

2 (i) State or imply that \(4^x = y^2 (= 2^{2x})\)
(ii) Carry out recognisable solution method for a quadratic equation in \(y\)
\begin{align*}
\text{Obtain } y = 3 \text{ and } y = \frac{1}{3} \text{ from } 3y^2 - 10y + 3 = 0
\end{align*}
Use logarithmic method to solve an equation of the form \(2^x = k\), where \(k > 0\)
State answer 1.58
State answer \(-1.58\) \(\text{(A1 \checkmark if } \pm \text{1.59)}\)

3 (i) Substitute \(x = \frac{3}{2}\) and equate to zero
\begin{align*}
\text{Obtain answer } a = -3
\end{align*}
(ii) At any stage, state that \(x = \frac{3}{2}\) is a solution
\begin{align*}
\text{\textit{EITHER: } Attempt division by } 2x - 3 \text{ reaching a partial quotient of } 2x^2 + kx
\end{align*}
\begin{align*}
\text{Obtain quadratic factor } 2x^2 + 3x + 1
\end{align*}
\begin{align*}
\text{Obtain solutions } x = -1 \text{ and } x = -\frac{1}{2}
\end{align*}
\begin{align*}
\text{\textit{OR: } Obtain solution } x = -1 \text{ by trial and error or inspection}
\end{align*}
\begin{align*}
\text{Obtain solution } x = -\frac{1}{2} \text{ similarly}
\end{align*}
\begin{align*}
\text{[If an attempt at the quadratic factor is made by inspection, the M1 is earned if it reaches an unknown factor of } 2x^2 + bx + c \text{ and an equation in } b \text{ and/or } c \text{.}]
\end{align*}

4 (i) Use \(\tan(A \pm B)\) formula to express LHS in terms of \(\tan x\)
\begin{align*}
\text{Obtain } \frac{\tan x + 1}{1 - \tan x} \text{, or equivalent}
\end{align*}
\begin{align*}
\text{Make relevant use of the tan 2A formula}
\end{align*}
\begin{align*}
\text{Obtain given answer correctly}
\end{align*}
(ii) State or imply \(2x = \tan^{-1}(2/2)\)
\begin{align*}
\text{Obtain answer } x = 22\frac{1}{2}^\circ
\end{align*}
\begin{align*}
\text{Obtain answer } x = 112\frac{1}{2}\circ \text{ and no others in range}
\end{align*}
5  (i) Obtain area of shaded segment in terms of $r$ and $\alpha$, e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2 \sin \alpha$

Equate area of shaded segment to $\frac{1}{6}(\pi - \sin x)$, or equivalent

Obtain given answer correctly

(ii) Consider sign of $x - \sin x - \frac{1}{3}\pi$ at $x = \frac{1}{2}\pi$ and $x = \frac{2}{3}\pi$, or equivalent

Complete the argument correctly with appropriate calculations

(iii) Use the iterative formula correctly at least once

Obtain final answer 1.97

Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.965, 1.975)

6  (i) Use quotient or product rule

Obtain derivative in any correct form, e.g. $e^{2x}\left(\frac{2}{x} - \frac{1}{x^2}\right)$

Equate derivative to zero and solve for $x$

Obtain $x = \frac{1}{2}$

Obtain $y = 2e$ (or 5.44)

Show or imply correct ordinates 7.389..., 13.390..., 27.299...

Use correct formula, or equivalent, with $\Delta h = 0.5$ and three ordinates

Obtain answer 15.4 with no errors seen

(ii) Justify the statement that the rule gives an over-estimate

7  (i) Obtain derivative of the form $k\sec^2 2x$, where $k = 2$ or $k = 1$

Obtain correct derivative $2\sec^2 2x$

(ii) State or imply the indefinite integral is $\frac{1}{k}\tan 2x$, where $k = 2$ or $k = 1$

Substitute limits correctly

Obtain given answer $\frac{1}{2}\sqrt{3}$

Use $\tan^2 2x = \sec^2 2x - 1$ and attempt to integrate both terms, or equivalent

Substitute limits in indefinite integral of the form $\frac{1}{k}\tan 2x - x$, where $k = 2$ or $k = 1$

Obtain answer $\frac{1}{2}\sqrt{3} - \frac{1}{6}\pi$, or equivalent

(iii) State that the integrand is equivalent to $\frac{1}{2}\sec^2 2x$

Obtain answer $\frac{1}{4}\sqrt{3}$