NOVEMBER 2002

GCE Advanced Level
GCE Advanced Subsidiary Level

<table>
<thead>
<tr>
<th>MARK SCHEME</th>
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<tbody>
<tr>
<td>MAXIMUM MARK: 50</td>
</tr>
<tr>
<td>SYLLABUS/COMPONENT : 9709/5, 8719/5</td>
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<tr>
<td>MATHEMATICS (Mechanics 2)</td>
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</tbody>
</table>

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate
1

\( r = 4\text{cm} \)

Uses \( v = wr \)

Speed is \( 20\text{cm s}^{-1} \) (FT if \( r = \frac{1}{3} \text{ candidate paro, distance from } B \))

B1

M1

A1 3

2

(i) Takes moments about \( B \)

\[ T \cos 60^\circ \times 2 = 10g \times 1 \]

Obtains tension as 100 N

A1 2

(ii) Uses Hooke's Law (for expression in \( x \) or \( L \) on \( u_y \))

Obtains 100 = 200(3 - \( L \)/L or 100 = 200 \( x \)/(3 - \( x \))

A1 \( \text{ft} \)

Obtains natural length as 2 m

A1 3

3

(i) \( x = 10t, y = -5t^2 \)

Eliminates \( t \) to find an equation in \( x \) and \( y \) (allow if candidate observes the general trajectory equation)

Obtains \( y = -x^2/20 \) (allow \( y = -5t^2 \) for putting \( x \) as \( y \) in traj. equation)

A1 3

(ii) Uses \( \tan \theta = \frac{dy}{dx} \) or \( \tan \theta = \frac{\dot{y}}{\dot{x}} \)

Obtains \( x = 30 \) when \( y = -45 \), or \( t = 3 \) when \( y = -45 \) or \( x = 15 \) and \( \dot{x} = (x)30 \)

A1

Obtains angle as 108.4° (108.435) or 71.6° (71.565)

A1 3

4

\( a = 4^2/0.8 \) [= 20]

Uses Newton’s 2nd law horizontally to obtain a 3 term equation

M1

Obtains \( T_P + T_Q \cos 30^\circ = 0.5 \times 20 \)

\[ T_P + T_Q = \frac{20}{\sqrt{3}} \]

A1 \( \text{ft} \)

Resolves forces vertically to obtain a 3 term equation

M1

Obtains \( T_P \cos 60^\circ = T_Q \cos 60^\circ + 5 \)

\[ T_P - T_Q = 10 \]

A1

Alternatively for the above 4 marks

Uses Newton’s 2nd law perpendicular to \( BQ \) to obtain a 3 term equation

M2

Obtains \( T_P \cos 30^\circ - 0.5g \cos 30^\circ = 0.5 \times 20 \cos 60^\circ \)

\[ T_P = 5 + \frac{10}{\sqrt{3}} \]

A2 \( \text{ft} \)

[SR Allow A1 with 1 sign or trigonometric error]

Obtains tension in \( PB \) as 10.8 N (10.7735)

A1 6

NB Use of equal tensions can score B, M1, A1, M1, A1, or more.
<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>GPE = 0.075g(d sin30°) or 0.075g(d + x)sin30°</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>EPE = 1.5(d - 2)^2/2x2 or 1.5x^2/2x2</td>
<td>B1</td>
</tr>
</tbody>
</table>
|   |     | Uses the principle of conservation of energy to form an equation with GPE and EPE terms \[
\frac{3}{8}d = \frac{3}{8}(d - 2)^2 \text{ or } \frac{3}{8}(2 + x) = \frac{3}{8}x^2 \]
|   |     | Attempts to solve a quadratic equation in d \[(d - 1)(d - 4) = 0\] or attempts to solve a quadratic equation in x and uses \[d = x + 2\]
|   |     | \[(x + 1)(x - 2) = 0 \text{ and } d = 2 + 2\] | M1* |
|   |     | Obtains distance as 4m                       | A1 |
|   | (ii)| Obtains the tension at the lowest point as 1.5 N ft for 1.5(d - 2)/2 | B1 ft |
|   |     | Uses Newton’s 2\(^{nd}\) law to obtain a 3 term equation | M1 |
|   |     | Obtains 1.5 - 0.075g sin 30° = 0.075\(a\)    | A1 |
|   |     | Obtains acceleration as 15ms\(^{-2}\)          | A1  |


6 (i) Uses Newton’s 2\textsuperscript{nd} law and \( a = v \frac{dv}{dx} \), and attempts to integrate

\[(1/10) v \frac{dv}{dx} = -v \cdot 200\]

\[v = -x/20 \quad (+C)
\]

Uses \( v(0) = 5 \) to find \( C \).  

Obtains \( v = -x/20 + 5 \)  

\[\text{M1}^* \]

\[\text{A1} \]

\[\text{M1 dep} \]

\[\text{A1} \quad 4 \]

(ii) Uses \( v = dx/dt \), separates the variables and integrates

\[
\left[ \int \frac{1}{100 - x} \, dx = \int \frac{1}{20} \, dt \right]
\]

Obtains \( \ln(100 - x) = -t/20 + C \)

\[\text{M1}^\# \]

Uses \( x = 0 \) when \( t = 0 \) to obtain \( t = 20[\ln 100 - \ln(100 - x)] \).

ft only if the term in \( x \) is logarithmic

\[\text{M1 dep} \]

\[\text{A1} \quad \text{ft} \]

For taking anti-logarithms throughout the equation \( x = 100(1 - e^{-20t}) \)

\[\text{M1 dep} \]

Obtains \( x = 100(1 - e^{-20t}) \)  

\[\text{A1} \quad 5 \]

Alternatively for the above 9 marks

Uses Newton’s 2\textsuperscript{nd} law with \( a = dv/dt \), separates the variables and integrates

\[
\left[ \int \frac{1}{v} \, dv = -\int \frac{1}{20} \, dt \right]
\]

Obtains \( \ln v = -t/20 + C \)

Uses \( v = 5 \) when \( t = 0 \) to obtain \( t = 20[\ln 5 - \ln v] \)

ft only if the term in \( v \) is logarithmic

For taking anti-logarithms throughout the equation \( v = 5e^{-20t} \)

\[\text{M1 dep} \]

Uses \( v = dx/dt \) and integrates

\[
x = 5e^{-20t} \cdot dt
\]

Obtains \( x = -100e^{-20t} + C \)

Uses \( x = 0 \) when \( t = 0 \) to obtain \( x = 100(1 - e^{-20t}) \) and \( v = 5e^{-20t} \) to obtain an

equation in \( x \) and \( v \)

\[\text{M1 dep} \]

Obtains \( v = -x/20 + 5 \)

\[\text{A1} \]

(iii) \( x = 100(1 - e^{-20t}) \) and \( e^{-20t} \) is \text{+ve} for all \( t \Rightarrow x < 100 \)  

\[\text{B1} \quad 1 \]

\[\text{NB} \quad \text{If } v \text{ is solved as in scheme and then } v \text{ is solved using}
\]

\[\text{the alternative method, the 5 marks awarded from the}
\]

\[\text{alternative method and the 3 marks from the}
\]

\[\text{alternative method, } v \text{ is solved using}
\]

\[\text{the alternative method and the integral is solved for } v \text{ from the}
\]

\[\text{alternative method.} \]
<table>
<thead>
<tr>
<th></th>
<th>Uses ((A_1 \pm A_2)x = A_1x_1 \pm A_2x_2) to find (x)</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([25\times 5 + 15\times 5)x = 25\times 5\times 12.5 + 15\times 5\times 2.5]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Obtains (x = 8.75)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Uses ((A_1 \pm A_2)y = A_1y_1 \pm A_2y_2) to find (y)</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>([25\times 5 + 15\times 5)y = 25\times 5\times 2.5 + 15\times 5\times 12.5]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Obtains (y = 6.25)</td>
<td>A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>States or obtains (\mu = \tan \alpha) for prism on point of sliding</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>States or obtains (\tan \alpha \leq x/y) for prism not toppled</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Eliminates (\tan \alpha) from (\mu = \tan \alpha) and (\tan \alpha \leq x/y), and substitutes for (x) and (y) ([\mu &lt; 8.75/6.25])</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Coefficient of friction is less than (7/5)((\text{convincing explanation for inequality}))</td>
<td>A1</td>
</tr>
<tr>
<td>(iii)</td>
<td>States or obtains (\tan \beta = y/x) for prism on point of toppling</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>States or obtains (\mu &gt; \tan \beta) for prism not sliding (or on the point of sliding)</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Eliminates (\tan \beta) from (\tan \beta = y/x) and (\mu &gt; \tan \beta), and substitutes for (x) and (y) ([\mu &gt; 6.25/8.75]) to obtain the least value of the coefficient of friction as (5/7)((\text{convincing explanation for inequality}))</td>
<td>A1</td>
</tr>
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</table>