Solve the equation \(3e^{2x} - 82e^x + 27 = 0\), giving your answers in the form \(k \ln 3\). 

\[ \text{[5]} \]
The variables $x$ and $y$ satisfy the equation $y = A \times B^{\ln x}$, where $A$ and $B$ are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (2.2, 4.908) and (5.9, 11.008), as shown in the diagram. Find the values of $A$ and $B$ correct to 2 significant figures. [5]
Without using a calculator, find the exact value of \( \int_{0}^{2} 4e^{-x}(e^{3x} + 1) \, dx \). [5]
The diagram shows the curve with equation \( y = \frac{5 \ln x}{2x + 1} \). The curve crosses the \( x \)-axis at the point \( P \) and has a maximum point \( M \).

(i) Find the gradient of the curve at the point \( P \). [3]
(ii) Show that the $x$-coordinate of the point $M$ satisfies the equation $x = \frac{x + 0.5}{\ln x}$.

(iii) Use an iterative formula based on the equation in part (ii) to find the $x$-coordinate of $M$ correct to 4 significant figures. Show the result of each iteration to 6 significant figures.
The parametric equations of a curve are
\[ x = 2 \cos 2\theta + 3 \sin \theta, \quad y = 3 \cos \theta \]
for \(0 < \theta < \frac{1}{2}\pi\).

(i) Find the gradient of the curve at the point for which \(\theta = 1\) radian.
(ii) Find the value of \( \sin \theta \) at the point on the curve where the tangent is parallel to the \( y \)-axis. [3]
The cubic polynomial \( f(x) \) is defined by

\[
f(x) = x^3 + ax^2 + 14x + a + 1,
\]

where \( a \) is a constant. It is given that \( (x + 2) \) is a factor of \( f(x) \).

(i) Use the factor theorem to find the value of \( a \) and hence factorise \( f(x) \) completely. [5]
(ii) Hence, without using a calculator, solve the equation \( f(2x) = 3f(x) \). [4]
7 (i) Express $5 \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of $\alpha$ correct to 4 decimal places. [3]

(ii) Using your answer from part (i), solve the equation

$$5 \cot \theta - 4 \cosec \theta = 2$$

for $0 < \theta < 2\pi$. [5]
(iii) Find \[
\int \frac{1}{\left(5 \cos \frac{1}{2}x - 2 \sin \frac{1}{2}x\right)^2} \, dx.
\] [3]