Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER  CANDIDATE NUMBER

MATHEMATICS  9709/11
Paper 1 Pure Mathematics 1 (P1)  May/June 2018
1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of 19 printed pages and 1 blank page.
1  (i) Find the first three terms in the expansion, in ascending powers of $x$, of $(1 - 2x)^5$. [2]

(ii) Given that the coefficient of $x^2$ in the expansion of $(1 + ax + 2x^2)(1 - 2x)^5$ is 12, find the value of the constant $a$. [3]
A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the $x$-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the $y$-coordinate when $x = 1$. [4]
A curve is such that \( \frac{dy}{dx} = \frac{12}{(2x + 1)^2} \). The point (1, 1) lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]
(i) Prove the identity \((\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta) = \sin^3 \theta + \cos^3 \theta\). \[3\]
(ii) Hence solve the equation \((\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta) = 3 \cos^3 \theta\) for \(0^\circ \leq \theta \leq 360^\circ\). [3]
The diagram shows a kite $OABC$ in which $AC$ is the line of symmetry. The coordinates of $A$ and $C$ are $(0, 4)$ and $(8, 0)$ respectively and $O$ is the origin.

(i) Find the equations of $AC$ and $OB$.  \[4\]
(ii) Find, by calculation, the coordinates of $B$. [3]
The diagram shows a circle with centre $O$ and radius $r$ cm. The points $A$ and $B$ lie on the circle and $AT$ is a tangent to the circle. Angle $AOB = \theta$ radians and $OBT$ is a straight line.

(i) Express the area of the shaded region in terms of $r$ and $\theta$. \[3\]
(ii) In the case where \( r = 3 \) and \( \theta = 1.2 \), find the perimeter of the shaded region.
Relative to an origin \( O \), the position vectors of the points \( A \), \( B \) and \( C \) are given by

\[
\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}.
\]

(i) Find \( \overrightarrow{AC} \). \[1\]

(ii) The point \( M \) is the mid-point of \( AC \). Find the unit vector in the direction of \( \overrightarrow{OM} \). \[3\]
(iii) Evaluate $\vec{AB}, \vec{AC}$ and hence find angle $BAC$. [4]
8 (a) A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]
(b) The $n$th term of a progression is $p + qn$, where $p$ and $q$ are constants, and $S_n$ is the sum of the first $n$ terms.

(i) Find an expression, in terms of $p$, $q$ and $n$, for $S_n$. [3]

(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of $p$ and $q$. [2]
Functions $f$ and $g$ are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \frac{1}{2}x - 2,$$

$$g : x \mapsto 4 + x - \frac{1}{2}x^2.$$

(i) Find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$.

(ii) Find the set of values of $x$ for which $f(x) > g(x)$.
(iii) Find an expression for \( fg(x) \) and deduce the range of \( fg \). [4]

The function \( h \) is defined by \( h : x \mapsto 4 + x - \frac{1}{2}x^2 \) for \( x \geq k \).

(iv) Find the smallest value of \( k \) for which \( h \) has an inverse. [2]
The curve with equation \( y = x^3 - 2x^2 + 5x \) passes through the origin.

(i) Show that the curve has no stationary points. \([3]\]

(ii) Denoting the gradient of the curve by \( m \), find the stationary value of \( m \) and determine its nature. \([5]\)
(iii) Showing all necessary working, find the area of the region enclosed by the curve, the $x$-axis and the line $x = 6$. [4]