1 The time taken for a particular type of paint to dry was measured for a sample of 150 randomly chosen points on a wall. The sample mean was 192.4 minutes and an unbiased estimate of the population variance was 43.6 minutes$^2$. Find a 98% confidence interval for the mean drying time. [3]

2 In the past, the mean annual crop yield from a particular field has been 8.2 tonnes. During the last 16 years, a new fertiliser has been used on the field. The mean yield for these 16 years is 8.7 tonnes. Assume that yields are normally distributed with standard deviation 1.2 tonnes. Carry out a test at the 5% significance level of whether the mean yield has increased. [5]

3 1% of adults in a certain country own a yellow car.

(i) Use a suitable approximating distribution to find the probability that a random sample of 240 adults includes more than 2 who own a yellow car. [4]

(ii) Justify your approximation. [2]

4 The number of sightings of a golden eagle at a certain location has a Poisson distribution with mean 2.5 per week. Drilling for oil is started nearby. A naturalist wishes to test at the 5% significance level whether there are fewer sightings since the drilling began. He notes that during the following 3 weeks there are 2 sightings.

(i) Find the critical region for the test and carry out the test. [5]

(ii) State the probability of a Type I error. [1]

(iii) State why the naturalist could not have made a Type II error. [1]

5 The time, $T$ minutes, taken by people to complete a test has probability density function given by

$$f(t) = \begin{cases} k(10t - t^2) & \text{if } 5 \leq t \leq 10, \\ 0 & \text{otherwise}, \end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{3}{250}$. [3]

(ii) Find $E(T)$. [3]

(iii) Find the probability that a randomly chosen value of $T$ lies between $E(T)$ and the median of $T$. [3]

(iv) State the greatest possible length of time taken to complete the test. [1]
6 \( X \) and \( Y \) are independent random variables with distributions Po(1.6) and Po(2.3) respectively.

(i) Find \( P(X + Y = 4) \).

A random sample of 75 values of \( X \) is taken.

(ii) State the approximate distribution of the sample mean, \( \bar{X} \), including the values of the parameters.

(iii) Hence find the probability that the sample mean is more than 1.7.

(iv) Explain whether the Central Limit theorem was needed to answer part (ii).

7 Bags of sugar are packed in boxes, each box containing 20 bags. The masses of the boxes, when empty, are normally distributed with mean 0.4 kg and standard deviation 0.01 kg. The masses of the bags are normally distributed with mean 1.02 kg and standard deviation 0.03 kg.

(i) Find the probability that the total mass of a full box of 20 bags is less than 20.6 kg.

(ii) Two full boxes are chosen at random. Find the probability that they differ in mass by less than 0.02 kg.