READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The length of time, in minutes, taken by people to complete a task has mean 53.0 and standard deviation 6.2. Find the probability that the mean time taken to complete the task by a random sample of 50 people is more than 51 minutes. 

Jacques is a chef. He claims that 90% of his customers are satisfied with his cooking. Marie suspects that the true percentage is lower than 90%. She asks a random sample of 15 of Jacques’ customers whether they are satisfied. She then performs a hypothesis test of the null hypothesis \( p = 0.9 \) against the alternative hypothesis \( p < 0.9 \), where \( p \) is the population proportion of customers who are satisfied. She decides to reject the null hypothesis if fewer than 12 customers are satisfied.

(i) In the context of the question, explain what is meant by a Type I error.

(ii) Find the probability of a Type I error in Marie’s test.

(i) Give a reason for using a sample rather than the whole population in carrying out a statistical investigation.

(ii) Tennis balls of a certain brand are known to have a mean height of bounce of 64.7 cm, when dropped from a height of 100 cm. A change is made in the manufacturing process and it is required to test whether this change has affected the mean height of bounce. 100 new tennis balls are tested and it is found that their mean height of bounce when dropped from a height of 100 cm is 65.7 cm and the unbiased estimate of the population variance is 15 cm\(^2\).

(a) Calculate a 95% confidence interval for the population mean.

(b) Use your answer to part (ii)(a) to explain what conclusion can be drawn about whether the change has affected the mean height of bounce.

At a certain company, computer faults occur randomly and at a constant mean rate. In the past this mean rate has been 2.1 per week. Following an update, the management wish to determine whether the mean rate has changed. During 20 randomly chosen weeks it is found that 54 computer faults occur. Use a suitable approximation to test at the 5% significance level whether the mean rate has changed.

Each box of Fruity Flakes contains \( X \) grams of flakes and \( Y \) grams of fruit, where \( X \) and \( Y \) are independent random variables, having distributions \( N(400, 50) \) and \( N(100, 20) \) respectively. The weight of each box, when empty, is exactly 20 grams. A full box of Fruity Flakes is chosen at random.

(i) Find the probability that the total weight of the box and its contents is less than 530 grams.

(ii) Find the probability that the weight of flakes in the box is more than 4.1 times the weight of fruit in the box.
3

6 At a certain shop the demand for hair dryers has a Poisson distribution with mean 3.4 per week.

(i) Find the probability that, in a randomly chosen two-week period, the demand is for exactly 5 hair dryers. [3]

(ii) At the beginning of a week the shop has a certain number of hair dryers for sale. Find the probability that the shop has enough hair dryers to satisfy the demand for the week if

(a) they have 4 hair dryers in the shop. [2]

(b) they have 5 hair dryers in the shop. [2]

(iii) Find the smallest number of hair dryers that the shop needs to have at the beginning of a week so that the probability of being able to satisfy the demand that week is at least 0.9. [3]

7 (a)

The diagram shows the graph of the probability density function of a variable $X$. Given that the graph is symmetrical about the line $x = 1$ and that $P(0 < X < 2) = 0.6$, find $P(X > 0)$. [2]

(b) A flower seller wishes to model the length of time that tulips last when placed in a jug of water. She proposes a model using the random variable $X$ (in hundreds of hours) with probability density function given by

$$f(x) = \begin{cases} k(2.25 - x^2) & 0 \leq x \leq 1.5, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{4}{9}$. [3]

(ii) Use this model to find the mean number of hours that a tulip lasts in a jug of water. [4]

The flower seller wishes to create a similar model for daffodils. She places a large number of daffodils in jugs of water and the longest time that any daffodil lasts is found to be 290 hours.

(iii) Give a reason why $f(x)$ would not be a suitable model for daffodils. [1]

(iv) The flower seller considers a model for daffodils of the form

$$g(x) = \begin{cases} c(a^2 - x^2) & 0 \leq x \leq a, \\ 0 & \text{otherwise,} \end{cases}$$

where $a$ and $c$ are constants. State a suitable value for $a$. (There is no need to evaluate $c$.) [1]