1 A small ball is projected with speed 16 m s\(^{-1}\) at an angle of 45\(^\circ\) above the horizontal from a point on horizontal ground. Calculate the period of time, before the ball lands, for which the speed of the ball is less than 12 m s\(^{-1}\). [4]

A uniform wire has the shape of a semicircular arc, with diameter AB of length 0.8 m. The wire is attached to a vertical wall by a smooth hinge at A. The wire is held in equilibrium with AB inclined at 70\(^\circ\) to the upward vertical by a light string attached to B. The other end of the string is attached to the point C on the wall 0.8 m vertically above A. The tension in the string is 15 N (see diagram).

(i) Show that the horizontal distance of the centre of mass of the wire from the wall is 0.463 m, correct to 3 significant figures. [3]

(ii) Calculate the weight of the wire. [2]

3 A particle P of mass 0.4 kg is released from rest at a point O on a smooth plane inclined at 30\(^\circ\) to the horizontal. When the displacement of P from O is x m down the plane, the velocity of P is \(v\) m s\(^{-1}\). A force of magnitude 0.8e\(^{-x}\) N acts on P up the plane along the line of greatest slope through O.

(i) Show that \(\frac{dv}{dx} = 5 - 2e^{-x}\). [2]

(ii) Find \(v\) when \(x = 0.6\). [4]
A uniform solid cone has base radius 0.4 m and height 4.4 m. A uniform solid cylinder has radius 0.4 m and weight equal to the weight of the cone. An object is formed by attaching the cylinder to the cone so that the base of the cone and a circular face of the cylinder are in contact and their circumferences coincide. The object rests in equilibrium with its circular base on a plane inclined at an angle of 20° to the horizontal (see diagram).

(i) Calculate the least possible value of the coefficient of friction between the plane and the object. [2]

(ii) Calculate the greatest possible height of the cylinder. [4]

5 A particle is projected at an angle of θ° below the horizontal from a point at the top of a vertical cliff 26 m high. The particle strikes horizontal ground at a distance 8 m from the foot of the cliff 2 s after the instant of projection. Find

(i) the speed of projection of the particle and the value of θ, [6]

(ii) the direction of motion of the particle immediately before it strikes the ground. [3]

[Questions 6 and 7 are printed on the next page.]
A light inextensible string passes through a small smooth bead $B$ of mass 0.4 kg. One end of the string is attached to a fixed point $A$ 0.4 m above a fixed point $O$ on a smooth horizontal surface. The other end of the string is attached to a fixed point $C$ which is vertically below $A$ and 0.3 m above the surface. The bead moves with constant speed on the surface in a circle with centre $O$ and radius 0.3 m (see diagram).

(i) Given that the tension in the string is 2 N, calculate

(a) the angular speed of the bead, [3]

(b) the magnitude of the contact force exerted on the bead by the surface. [2]

(ii) Given instead that the bead is about to lose contact with the surface, calculate the speed of the bead. [4]

A particle $P$ is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 12 N. The other end of the string is attached to a fixed point $O$ on a smooth plane inclined at an angle of 30° to the horizontal. $P$ rests in equilibrium on the plane, 1.6 m from $O$.

(i) Calculate the mass of $P$. [2]

A particle $Q$, with mass equal to the mass of $P$, is projected up the plane along a line of greatest slope. When $Q$ strikes $P$ the two particles coalesce. The combined particle remains attached to the string and moves up the plane, coming to instantaneous rest after moving 0.2 m.

(ii) Show that the initial kinetic energy of the combined particle is 1 J. [4]

The combined particle subsequently moves down the plane.

(iii) Calculate the greatest speed of the combined particle in the subsequent motion. [5]