READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 50.
1. Given that $5^x = 7^y$, use logarithms to find the value of $\frac{x}{y}$ correct to 4 significant figures. [3]

2. (i) Find the quotient and remainder when $2x^3 - 7x^2 - 9x + 3$ is divided by $x^2 - 2x + 5$. [3]
(ii) Hence find the values of the constants $p$ and $q$ such that $x^2 - 2x + 5$ is a factor of $2x^3 - 7x^2 + px + q$. [2]

3. (i) Solve the equation $|3u + 1| = |2u - 5|$. [3]
(ii) Hence solve the equation $|3 \cot x + 1| = |2 \cot x - 5|$ for $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [2]

4. (i) Show that $\sin(\theta + 60^\circ) + \sin(\theta + 120^\circ) \equiv (\sqrt{3}) \cos \theta$. [3]
(ii) Hence
   (a) find the exact value of $\sin 105^\circ + \sin 165^\circ$. [2]
   (b) solve the equation $\sin(\theta + 60^\circ) + \sin(\theta + 120^\circ) = \sec \theta$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

5. The equation of a curve is $y = 6xe^{\frac{1}{3}x}$. At the point on the curve with $x$-coordinate $p$, the gradient of the curve is 40.
   (i) Show that $p = 3 \ln \left(\frac{20}{p + 3}\right)$. [4]
   (ii) Show by calculation that $3.3 < p < 3.5$. [2]
   (iii) Use an iterative formula based on the equation in part (i) to find the value of $p$ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]
6 (a) Find \( \int \frac{4 + e^x}{2e^{2x}} \, dx \). \[3\]

(b) Without using a calculator, find \( \int_{2}^{10} \frac{1}{2x + 5} \, dx \), giving your answer in the form \( \ln k \). \[4\]

(c)

The diagram shows the curve \( y = \log_{10}(x + 2) \) for \( 0 \leq x \leq 6 \). The region bounded by the curve and the lines \( x = 0 \), \( x = 6 \) and \( y = 0 \) is denoted by \( R \). Use the trapezium rule with 2 strips to find an estimate of the area of \( R \), giving your answer correct to 1 decimal place. \[3\]

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The diagram shows the curve with parametric equations

\[
x = 2 - \cos t, \quad y = 1 + 3 \cos 2t,
\]

for \( 0 < t < \pi \). The minimum point is \( M \) and the curve crosses the \( x \)-axis at points \( P \) and \( Q \).

(i) Show that \( \frac{dy}{dx} = -12 \cos t \). \[4\]

(ii) Find the coordinates of \( M \). \[2\]

(iii) Find the gradient of the curve at \( P \) and at \( Q \). \[4\]