Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.
1. \( f : x \mapsto 10 - 3x, \ g : x \mapsto \frac{10}{3 - 2x}, \)

\[
\begin{align*}
ff(x) &= 10 - 3(10 - 3x) \\
gf(2) &= \frac{10}{3 - 2(10 - 3(2))} = -2
\end{align*}
\]

\( x = 2 \)

B1 Correct unsimplified expression

B1 Correct unsimplified expression with 2 in for \( x \)

B1

[3]

2. \( f'(x) = \frac{8}{(5 - 2x)^2} \)

\[
\begin{align*}
f(x) &= \frac{8(5 - 2x)^{-1}}{-1} + -2 (+c) \\
\text{Uses } x &= 2, y = 7, \ \text{and } c = 3
\end{align*}
\]

B1 Correct without \( \div -2 \)

B1 An attempt at integration \( \div -2 \)

M1 Substitution of correct values into an integral to find \( c \)

A1

[4]

3. \( \overrightarrow{OA} = 2i - 5j - 2k \) and \( \overrightarrow{OB} = 4i - 4j + 2k. \)

\( \overrightarrow{AB} = 2i + j + 4k \) or \( \overrightarrow{AC} = 4i + 2j + 8k \)

\( \overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 6i - 3j + 6k \)

B1

M1 correct method for \( \overrightarrow{OC} \)

OR

\[
\begin{bmatrix}
2 \\
1 \\
4
\end{bmatrix}
= \begin{bmatrix}
x - 4 \\
y + 4 \\
z - 2
\end{bmatrix}
\]

M1

OR

\[
\begin{align*}
\overrightarrow{OC} &= \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
6 \\
-3 \\
6
\end{bmatrix}
\end{align*}
\]

M1

\[
\begin{align*}
\overrightarrow{OB} - \overrightarrow{OA} &= \overrightarrow{OC} - \overrightarrow{OB} \\
\therefore \overrightarrow{OC} &= 2\overrightarrow{OB} - \overrightarrow{OA}
\end{align*}
\]

\[
\begin{bmatrix}
8 \\
-8 \\
4
\end{bmatrix} - \begin{bmatrix}
2 \\
-5 \\
-2
\end{bmatrix} = \begin{bmatrix}
6 \\
-3 \\
6
\end{bmatrix}
\]

M1 Divides by their mod of their \( \overrightarrow{OC} \)

A1 Correct unsimplified expression

[4]

Unit vector = (Their \( \overrightarrow{OC} \)) \div (Mod their \( \overrightarrow{OC} \))

\( = (6i - 3j + 6k) \div 9 \)
### Question 4

#### (i)

\[
\left( x - \frac{2}{3} \right)^6
\]

Term is \( \binom{6}{x} C_3 \times (-2)^3 = (-160) \times 160 = -160 \)

B1 \( \pm 160 \) seen anywhere [2]

#### (ii)

\[
\left( 2 + \frac{3}{x^2} \right) \left( x - \frac{2}{x} \right)^6
\]

Term in \( x^2 = \binom{6}{x} C_2 \times (-2)^2 x^2 = 60 \) (\( x^2 \))

B1 \( \pm 60 \) seen anywhere

Term independent of \( x \):

\( = 2 \times (\text{their} -160) + 3 \times (\text{their} 60) = -140 \)


### Question 5

#### (i)

\[
\tan \left( \frac{\pi}{3} \right) = \frac{AC}{2x} \quad \text{or} \quad \cos \left( \frac{\pi}{3} \right) = \frac{\sin \pi}{6} = \frac{2x}{AB}
\]

\( \rightarrow AC = 2\sqrt{3}x \) or \( AB = 4x \)

AM = \sqrt{13x^2}, \sqrt{13x}, 3.61x

B1 Either trig ratio

M1A1 Complete method. [3]

#### (ii)

\[
\tan (M\hat{A}C) = \frac{x}{\text{Their } AC}
\]

\( \theta = \frac{1}{6} \pi - \tan^{-1} \frac{1}{2} \sqrt{3} \text{AG} \)

M1 “Their AC” must be \( f(x) \), \( \left( M\hat{A}C \right) \neq \theta \).

A1 Justifies \( \frac{\pi}{6} \) and links MAC & \( \theta \) [2]

### Question 6

#### (i)

\[
PT = r \tan \alpha
\]

\[
QT = OT - OQ = \frac{r}{\cos \alpha} - r
\]

or \( \sqrt{r^2 + r^2 \tan^2 \alpha} - r \)

B1

Perimeter = sum of the 3 parts including \( r\alpha \) B1 [3]

#### (ii)

Area of triangle = \( \frac{1}{2} \times 10 \times 10 \tan \frac{\pi}{3} \)

Area of sector = \( \frac{1}{2} \times 10^2 \times \frac{1}{3} \pi \)

M1 Correct formula used, \( 50\sqrt{3}, 86.6 \)

Shaded region has area 34 (2sf)

M1 Correct formula used, \( \frac{50\pi}{3} \), 52.36 A1 [3]
7 (i) \[ \frac{1 + \cos \theta - 1 - \cos \theta}{1 - \cos \theta + 1 + \cos \theta} \equiv \frac{4}{\sin \theta \tan \theta} \]
\[ \text{LHS} = \frac{1 + 2c + c^2 - (1 - 2c + c^2)}{(1 - c)(1 + c)} \]
\[ = \frac{4c}{1 - e^2} \]
\[ = \frac{4c}{c^2} \]
\[ = \frac{4}{ts} \text{ AG} \]

Attempt at combining fractions. A1 for numerator. A1 denominator Essential step for award of A1

(ii) \[ \sin \theta \left( \frac{1 + \cos \theta - 1 - \cos \theta}{1 - \cos \theta + 1 + \cos \theta} \right) = 3. \]
\[ \rightarrow s \times \frac{4}{ts} = 3 \rightarrow t = \frac{4}{3} \]
\[ \theta = 53.1^\circ \text{ and } 233.1^\circ \]

Uses part (i) to eliminate “s” correctly. √ for $180^\circ + 1^\text{st}$ answer.

8

(i) A (0, 7), B (8, 3) and C (3k, k)

m of AB is $-\frac{1}{2}$ oe.
Eqn of AB is $y = -\frac{1}{2}x + 7$
Let $x = 3k$, $y = k$
k = 2.8 oe

OR
\[ \frac{7 - k}{0 - 3k} = \frac{3 - k}{8 - 3k} \]
\[ \rightarrow 20k = 56 \rightarrow k = 2.8 \]

OR
\[ \frac{7 - k}{0 - 3k} = \frac{7 - 3}{0 - 8} \]
\[ \rightarrow 20k = 56 \rightarrow k = 2.8 \]

Using $A,B$ or C to get an equation Using $C$ or $A,B$ in the equation Using $A$ or $B$ & in the equation

Using $A,B$ & C to equate gradients Simplifies to a linear or 3 term quadratic = 0.

Using $A,B$ and C to equate gradients Simplifies to a linear or 3 term quadratic = 0.
| (ii) | M(4, 5)  
Perpendicular gradient = 2.  
Perp bisector has eqn \( y - 5 = 2(x - 4) \)  
Let \( x = 3k, y = k \)  
k = \( \frac{3}{5} \) oe  
OR  
\( (0 - 3k)^2 + (7 - k)^2 = (8 - 3k)^2 + (3 - k)^2 \)  
\(-14k + 49 = 73 - 54k \rightarrow 40k = 24 \rightarrow k = 0.6\) | B1  
anywhere in (ii)  
M1  
Use of \( m_1m_2 = -1 \) soi  
M1  
Forming eqn using their M and their "perpendicular m"  
A1 |}

| 9 (i) (a) |  \( a + (n-1)d = 10 + 29\times2 \)  
\( = 68 \) | M1  
Use of \( n \)th term of an AP with \( a=\pm10, d=\pm2, n=30 \) or \( 29 \)  
A1  
Condone – 68 → 68 |}

|  | \( \frac{1}{2}n(20 + 2(n-1)) = 2000 \) or 0  
\( \rightarrow 2n^2 + 18n - 4000 = 0 \) oe \( (n=) 41 \) | M1  
Use of \( S_n \) formula for an AP with \( a=\pm10, d=\pm2 \) and equated to either 0 or 2000.  
A1  
Correct 3 term quadratic = 0.  
A1 |}

| 9 (i) (ii) | \( r = 1.1, \) oe  
Uses \( S_{30} = \frac{10\left(1.1^{30} - 1\right)}{1.1-1} (= 1645) \)  
Percentage lost = \( \frac{2000 - 1645}{2000} \times 100 \)  
\( = 17.75 \) | B1  
e.g. \( \frac{11}{10}, 110\% \)  
M1  
Use of \( S_n \) formula for a GP, \( a=\pm10, n=30. \)  
DM1  
Fully correct method for % left with “their 1645”  
A1  
allow 17.7 or 17.8. |}

| 10 | \( y = \frac{8}{x} + 2x \)  
(i)  
\( \frac{dy}{dx} = -8x^{-2} + 2 \)  
\( \frac{d^2y}{dx^2} = 16x^{-3} \)  
\( \int y^2dx = -64x^{-4} \text{ oe} + 32x \text{ oe} + \frac{4x^3}{3} \text{ oe } (+c) \)  
\( 3 \times B1 \) | B1  
un simplified ok  
B1  
un simplified ok  
B1 for each term – unsimplified ok |
(ii) \( \frac{dy}{dx} \) to 0 \( \rightarrow x = \pm 2 \)  
\( \rightarrow M(2, 8) \)  
Other turning point is \((-2, -8)\)  
If \( x = -2 \), \( \frac{d^2y}{dx^2} < 0 \)  
\( \therefore \) Maximum \( M(2, 8) \)  
M1 \( A1 \) \( A1 \) \[5\]  
Sets to 0 and attempts to solve  
Any pair of correct values \( A1 \)  
Second pair of values \( A1 \)  
Using their \( \frac{d^2y}{dx^2} \) if \( kx^3 \) and \( x < 0 \)  
A1 \[5\]  
Sets to 0 and attempts to solve  
Any pair of correct values \( A1 \)  
Second pair of values \( A1 \)  
Using their \( \frac{d^2y}{dx^2} \) if \( kx^3 \) and \( x < 0 \)  
A1 \[5\]  
Evidence of using limits 1&2 in their integral of \( y^2 \) (ignore \( \pi \)) \( A1 \) \[2\]  

| 11 | Sets to 0 and attempts to solve | A1 | Any pair of correct values | A1 | Second pair of values | A1 | Using their \( \frac{d^2y}{dx^2} \) if \( kx^3 \) and \( x < 0 \) | A1 | \[5\] | Evidence of using limits 1&2 in their integral of \( y^2 \) (ignore \( \pi \)) | A1 | \[2\] | \[3\] | \[3\] | \[1\] | Correct order of operations. | M1A0 \( \sqrt{4-x} + 3 \) | M1A0 \( \sqrt{x-4} + 3 \) | M1A0 | \( \sqrt{4-y} + 3 \) | M1A0 | \[2\] | \[2\] | \[2\] |

\[ f : x \mapsto 6x - x^2 - 5 \]

(i) \( 6x - x^2 - 5 \leq 3 \)  
\( \rightarrow x^2 - 6x + 8 \geq 0 \)  
\( \rightarrow x = 2, x = 4 \)  
\( x \leq 2, x \geq 4 \) \( \leq \) and/or \( \geq \) condone  
M1 \( A1 \) \[3\]  
\( \geq \) and/or \( \leq \) condone  
M1 \( A1 \) \[3\]  
\( \geq \) and/or \( \leq \) condone  
M1 \( A1 \) \[3\]  
\( \geq \) and/or \( \leq \) condone  
M1 \( A1 \) \[3\]  

(ii) Equate \( mx + c \) and \( 6x - x^2 - 5 \)  
Use of “\( b^2 - 4ac \)” \( 4c = m^2 - 12m + 16. AG \)  
OR \( \frac{dy}{dx} = 6 - 2x = m \rightarrow x = \left( \frac{6-m}{2} \right) \)  
M1 \( A1 \) \[3\]  
Equates \( \frac{dy}{dx} \) to \( m \) and rearrange \( m \left( \frac{6-m}{2} \right) + c = 6 \left( \frac{6-m}{2} \right) - \left( \frac{6-m}{2} \right)^2 - 5 \)  
M1 \( A1 \) \[3\]  
Equates \( mx + c \) and \( 6x - x^2 - 5 \) and substitutes for \( x \) \( 4c = m^2 - 12m + 16. AG \)  
M1 \( A1 \) \[3\]  

(iii) \( 6x - x^2 - 5 = 4 - (x - 3)^2 \)  
\[ B1 B1 \] \( 4 B1 - (x - 3)^2 B1 \) \[2\]  

(iv) \( k = 3 \)  
\[ B1^\text{\( \wedge \)} \] \( \text{for “} b \text{”} \). \[1\]  

(v) \( g^{-1}(x) = \sqrt{4-x} + 3 \)  
\[ M1 A1 \] \[2\]  
Correct order of operations. \( \pm \sqrt{4-x} + 3 \) \( M1A0 \)  
\( \sqrt{x-4} + 3 \) \( M1A0 \)  
\( \sqrt{4-y} + 3 \) \( M1A0 \)  
\[2\]  

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