READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 3 printed pages and 1 blank page.
The random variable $X$ has probability density function, $f$, as shown in the diagram, where $a$ is a constant. Find the value of $a$ and hence show that $E(X) = 0.943$ correct to 3 significant figures.  

Sami claims that he can read minds. He asks each of 50 people to choose one of the 5 letters A, B, C, D or E. He then tells each person which letter he believes they have chosen. He gets 13 correct. Sami says “This shows that I can read minds, because 13 is more than I would have got right if I were just guessing.”

(i) State null and alternative hypotheses for a test of Sami’s claim. 

(ii) Test at the 10% significance level whether Sami’s claim is justified. 

The daily times, in minutes, that Yu Ming takes showering, getting dressed and having breakfast are independent and have the distributions $N(9, 2.2^2)$, $N(8, 1.3^2)$ and $N(17, 2.6^2)$ respectively. The total daily time that Yu Ming takes for all three activities is denoted by $T$ minutes.

(i) Find the mean and variance of $T$. 

(ii) Yu Ming notes the value of $T$ on each day in a random sample of 70 days and calculates the sample mean. Find the probability that the sample mean is between 33 and 35. 

In the past, the time taken by vehicles to drive along a particular stretch of road has had mean 12.4 minutes and standard deviation 2.1 minutes. Some new signs are installed and it is expected that the mean time will increase. In order to test whether this is the case, the mean time for a random sample of 50 vehicles is found. You may assume that the standard deviation is unchanged.

(i) The mean time for the sample of 50 vehicles is found to be 12.9 minutes. Test at the 2.5% significance level whether the population mean time has increased. 

(ii) State what is meant by a Type II error in this context. 

(iii) State what extra piece of information would be needed in order to find the probability of a Type II error.
5 The masses, \( m \) grams, of a random sample of 80 strawberries of a certain type were measured and summarised as follows.

\[
n = 80 \quad \Sigma m = 4200 \quad \Sigma m^2 = 229000
\]

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Calculate a 98% confidence interval for the population mean. [3]

50 random samples of size 80 were taken and a 98% confidence interval for the population mean, \( \mu \), was found from each sample.

(iii) Find the number of these 50 confidence intervals that would be expected to include the true value of \( \mu \). [1]

6 A publishing firm has found that errors in the first draft of a new book occur at random and that, on average, there is 1 error in every 3 pages of a first draft. Find the probability that in a particular first draft there are

(i) exactly 2 errors in 10 pages, [2]

(ii) at least 3 errors in 6 pages, [3]

(iii) fewer than 50 errors in 200 pages. [4]

7 The independent variables \( X \) and \( Y \) are such that \( X \sim B(10, 0.8) \) and \( Y \sim Po(3) \). Find

(i) \( E(7X + 5Y - 2) \), [2]

(ii) \( Var(4X - 3Y + 3) \), [4]

(iii) \( P(2X - Y = 18) \). [4]