1 Solve the equation \( \ln(x + 4) = 2 \ln x + \ln 4 \), giving your answer correct to 3 significant figures. [4]

2 Solve the inequality \(|x - 2| > 2x - 3\). [4]

3 Solve the equation \( \cot 2x + \cot x = 3 \) for \( 0^\circ < x < 180^\circ \). [6]

4 The curve with equation \( y = \frac{e^{2x}}{4 + e^{3x}} \) has one stationary point. Find the exact values of the coordinates of this point. [6]

5 The parametric equations of a curve are

\[ x = a \cos^4 t, \quad y = a \sin^4 t, \]

where \( a \) is a positive constant.

(i) Express \( \frac{dy}{dx} \) in terms of \( t \). [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter \( t \) is

\[ x \sin^2 t + y \cos^2 t = a \sin^2 t \cos^2 t. \] [3]

(iii) Hence show that if the tangent meets the \( x \)-axis at \( P \) and the \( y \)-axis at \( Q \), then

\[ OP + OQ = a, \]

where \( O \) is the origin. [2]

6 It is given that \( \int_0^a x \cos x \, dx = 0.5 \), where \( 0 < a < \frac{1}{2} \pi \).

(i) Show that \( a \) satisfies the equation \( \sin a = \frac{1.5 - \cos a}{a} \). [4]

(ii) Verify by calculation that \( a \) is greater than 1. [2]

(iii) Use the iterative formula

\[ a_{n+1} = \sin^{-1} \left( \frac{1.5 - \cos a_n}{a_n} \right) \]

to determine the value of \( a \) correct to 4 decimal places, giving the result of each iteration to 6 decimal places. [3]
The number of micro-organisms in a population at time \( t \) is denoted by \( M \). At any time the variation in \( M \) is assumed to satisfy the differential equation

\[
\frac{dM}{dt} = k(\sqrt{M}) \cos(0.02t),
\]

where \( k \) is a constant and \( M \) is taken to be a continuous variable. It is given that when \( t = 0, \ M = 100. \)

(i) Solve the differential equation, obtaining a relation between \( M, k \) and \( t \). [5]

(ii) Given also that \( M = 196 \) when \( t = 50 \), find the value of \( k \). [2]

(iii) Obtain an expression for \( M \) in terms of \( t \) and find the least possible number of micro-organisms. [2]

The complex number \( 1 - i \) is denoted by \( u \).

(i) Showing your working and without using a calculator, express

\[
\frac{i}{u}
\]

in the form \( x + iy \), where \( x \) and \( y \) are real. [2]

(ii) On an Argand diagram, sketch the loci representing complex numbers \( z \) satisfying the equations \( |z - u| = |z| \) and \( |z - i| = 2. \) [4]

(iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part (ii). [3]

Two planes have equations \( x + 3y - 2z = 4 \) and \( 2x + y + 3z = 5. \) The planes intersect in the straight line \( l. \)

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line \( l. \) [6]

Let \( f(x) = \frac{11x + 7}{(2x - 1)(x + 2)^2}. \)

(i) Express \( f(x) \) in partial fractions. [5]

(ii) Show that \( \int_{-1}^{2} f(x) \, dx = \frac{1}{4} + \ln(\frac{3}{2}). \) [5]