1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. \[4\]

2 Use the trapezium rule with three intervals to find an approximation to

\[\int_{0}^{3} |3^x - 10| \, dx.\] \[4\]

3 Show that, for small values of $x^2$,

\[(1 - 2x^2)^{-2} - (1 + 6x^2)^{\frac{1}{2}} \approx kx^4,\]

where the value of the constant $k$ is to be determined. \[6\]

4 The equation of a curve is

\[y = 3 \cos 2x + 7 \sin x + 2.\]

Find the $x$-coordinates of the stationary points in the interval $0 \leq x \leq \pi$. Give each answer correct to 3 significant figures. \[7\]

5 (a) Find \[\int (4 + \tan^2 2x) \, dx.\] \[3\]

(b) Find the exact value of \[\int_{\frac{\pi}{4}}^{3\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} \, dx.\] \[5\]

6 The straight line $l_1$ passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line $l_2$ has equation $r = 7i + j + k + \mu(2i + 3j + 5k)$.

(i) Show that the lines $l_1$ and $l_2$ are skew. \[6\]

(ii) Find the acute angle between the direction of the line $l_2$ and the direction of the $x$-axis. \[3\]

7 Given that $y = 1$ when $x = 0$, solve the differential equation

\[\frac{dy}{dx} = 4x(3y^2 + 10y + 3),\]

obtaining an expression for $y$ in terms of $x$. \[9\]

8 The complex number $w$ is defined by $w = \frac{22 + 4i}{(2 - i)^2}$.

(i) Without using a calculator, show that $w = 2 + 4i$. \[3\]

(ii) It is given that $p$ is a real number such that $\frac{1}{4}\pi \leq \arg(w + p) \leq \frac{3}{4}\pi$. Find the set of possible values of $p$. \[3\]

(iii) The complex conjugate of $w$ is denoted by $w^*$. The complex numbers $w$ and $w^*$ are represented in an Argand diagram by the points $S$ and $T$ respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through $S$, $T$ and the origin. \[3\]
The diagram shows the curve \( y = x^2 e^{2-x} \) and its maximum point \( M \).

(i) Show that the \( x \)-coordinate of \( M \) is 2. [3]

(ii) Find the exact value of \( \int_0^2 x^2 e^{2-x} \, dx \). [6]

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The diagram shows part of the curve with parametric equations

\[ x = 2 \ln(t + 2), \quad y = t^3 + 2t + 3. \]

(i) Find the gradient of the curve at the origin. [5]

(ii) At the point \( P \) on the curve, the value of the parameter is \( p \). It is given that the gradient of the curve at \( P \) is \( \frac{1}{2} \).

(a) Show that \( p = \frac{1}{3p^2 + 2} - 2 \). [1]

(b) By first using an iterative formula based on the equation in part (a), determine the coordinates of the point \( P \). Give the result of each iteration to 5 decimal places and each coordinate of \( P \) correct to 2 decimal places. [4]