1 The weights, in grams, of a random sample of 8 packets of cereal are as follows.

250 248 255 244 259 250 242 258

Calculate unbiased estimates of the population mean and variance. [3]

2 Each day Samuel travels from A to B and from B to C. He then returns directly from C to A. The times, in minutes, for these three journeys have the independent distributions $N(20, 2^2)$, $N(18, 1.5^2)$ and $N(30, 1.8^2)$, respectively. Find the probability that, on a randomly chosen day, the total time for his two journeys from A to B and B to C is less than the time for his return journey from C to A. [5]

3 The number of calls per day to an enquiry desk has a Poisson distribution. In the past the mean has been 5. In order to test whether the mean has changed, the number of calls on a random sample of 10 days was recorded. The total number of calls was found to be 61. Use an approximate distribution to test at the 10% significance level whether the mean has changed. [5]

4 (i) The random variable $W$ has the distribution $\text{Po}(1.5)$. Find the probability that the sum of 3 independent values of $W$ is greater than 2. [3]

(ii) The random variable $X$ has the distribution $\text{Po}(\lambda)$. Given that $P(X = 0) = 0.523$, find the value of $\lambda$ correct to 3 significant figures. [2]

(iii) The random variable $Y$ has the distribution $\text{Po}(\mu)$, where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find $\mu$. [3]

5 Mahmoud throws a coin 400 times and finds that it shows heads 184 times. The probability that the coin shows heads on any throw is denoted by $p$.

(i) Calculate an approximate 95% confidence interval for $p$. [4]

(ii) Mahmoud claims that the coin is not fair. Use your answer to part (i) to comment on this claim. [1]

(iii) Mahmoud’s result of 184 heads in 400 throws gives an $\alpha$% confidence interval for $p$ with width 0.1. Calculate the value of $\alpha$. [4]

6 The time, $T$ hours, spent by people on a visit to a museum has probability density function

$$f(t) = \begin{cases} \frac{kt(16 - t^2)}{160} & 0 \leq t \leq 4, \\ 0 & \text{otherwise,} \end{cases}$$

where $k$ is a constant.

(i) Show that $k = \frac{1}{160}$. [3]

(ii) Calculate the probability that two randomly chosen people each spend less than 1 hour on a visit to the museum. [4]

(iii) Find the mean time spent on a visit to the museum. [3]
A researcher is investigating the actual lengths of time that patients spend with the doctor at their appointments. He plans to choose a sample of 12 appointments on a particular day.

(i) Which of the following methods is preferable, and why?

- Choose the first 12 appointments of the day.
- Choose 12 appointments evenly spaced throughout the day. [2]

Appointments are scheduled to last 10 minutes. The actual lengths of time, in minutes, that patients spend with the doctor may be assumed to have a normal distribution with mean $\mu$ and standard deviation 3.4. The researcher suspects that the actual time spent is more than 10 minutes on average. To test this suspicion, he recorded the actual times spent for a random sample of 12 appointments and carried out a hypothesis test at the 1% significance level.

(ii) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(iii) Given that the total length of time spent for the 12 appointments was 147 minutes, carry out the test. [5]

(iv) Give a reason why the Central Limit theorem was not needed in part (iii). [1]