1. A small sphere of mass 0.4 kg moves with constant speed 1.5 m s\(^{-1}\) in a horizontal circle inside a smooth fixed hollow cylinder of diameter 0.6 m. The axis of the cylinder is vertical, and the sphere is in contact with both the horizontal base and the vertical curved surface of the cylinder.

(i) Calculate the magnitude of the force exerted on the sphere by the vertical curved surface of the cylinder. [2]

(ii) Hence show that the magnitude of the total force exerted on the sphere by the cylinder is 5 N. [2]

2. A uniform semicircular lamina of radius 0.25 m has diameter \(AB\). It is freely suspended at \(A\) from a fixed point and hangs in equilibrium.

(i) Find the distance of the centre of mass of the lamina from the diameter \(AB\). [1]

(ii) Calculate the angle which the diameter \(AB\) makes with the vertical. [2]

The lamina is now held in equilibrium with the diameter \(AB\) vertical by means of a force applied at \(B\). This force has magnitude 6 N and acts at 45° to the upward vertical in the plane of the lamina.

(iii) Calculate the weight of the lamina. [3]

3. A particle \(P\) of mass 0.2 kg is attached to one end of a light elastic string of natural length 1.6 m and modulus of elasticity 18 N. The other end of the string is attached to a fixed point \(O\) which is 1.6 m above a smooth horizontal surface. \(P\) is placed on the surface vertically below \(O\) and then projected horizontally. \(P\) moves with initial speed 1.5 m s\(^{-1}\) in a straight line on the surface. Show that, when \(OP = 1.8\) m,

(i) \(P\) is at instantaneous rest, [3]

(ii) \(P\) is on the point of losing contact with the surface. [4]

4. A ball \(B\) is projected from a point \(O\) on horizontal ground at an angle of 40° above the horizontal. \(B\) hits the ground 1.8 s after the instant of projection. Calculate

(i) the speed of projection of \(B\), [2]

(ii) the greatest height of \(B\), [2]

(iii) the distance from \(O\) of the point at which \(B\) hits the ground. [2]
A block $B$ of mass 3 kg is attached to one end of a light elastic string of modulus of elasticity 70 N and natural length 1.4 m. The other end of the string is attached to a particle $P$ of mass 0.3 kg. $B$ is at rest 0.9 m from the edge of a horizontal table and the string passes over a small smooth pulley at the edge of the table. $P$ is released from rest at a point next to the pulley and falls vertically. At the first instant when $P$ is 0.8 m below the pulley and descending, $B$ is in limiting equilibrium with the part of the string attached to $B$ horizontal (see diagram).

(i) Calculate the speed of $P$ when $B$ is first in limiting equilibrium.  [5]

(ii) Find the coefficient of friction between $B$ and the table.  [3]

A uniform solid cone of height 0.6 m and mass 0.5 kg has its axis of symmetry vertical and its vertex $V$ uppermost. The semi-vertical angle of the cone is $60^\circ$ and the surface is smooth. The cone is fixed to a horizontal surface. A particle $P$ of mass 0.2 kg is connected to $V$ by a light inextensible string of length 0.4 m (see diagram).

(i) Calculate the height, above the horizontal surface, of the centre of mass of the cone with the particle.  [3]

$P$ is set in motion, and moves with angular speed 4 rad s$^{-1}$ in a circular path on the surface of the cone.

(ii) Show that the tension in the string is 1.96 N, and calculate the magnitude of the force exerted on $P$ by the cone.  [5]

(iii) Find the speed of $P$.  [1]

A particle $P$ of mass 0.5 kg moves in a straight line on a smooth horizontal surface. The velocity of $P$ is $v$ m s$^{-1}$ when the displacement of $P$ from $O$ is $x$ m. A single horizontal force of magnitude 0.16e$^x$ N acts on $P$ in the direction $OP$. The velocity of $P$ when it is at $O$ is 0.8 m s$^{-1}$.

(i) Show that $v = 0.8e^{\frac{1}{2}x}$.  [6]

(ii) Find the time taken by $P$ to travel 1.4 m from $O$.  [4]