1 Solve the equation $|x - 2| = \frac{1}{3}x$. \[3\]

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{x_n(x_n^3 + 100)}{2(x_n^3 + 25)},$$

with initial value $x_1 = 3.5$, converges to $\alpha$.

(i) Use this formula to calculate $\alpha$ correct to 4 decimal places, showing the result of each iteration to 6 decimal places. \[3\]

(ii) State an equation satisfied by $\alpha$ and hence find the exact value of $\alpha$. \[2\]

3 The variables $x$ and $y$ satisfy the equation $y = Ae^{-kx^2}$, where $A$ and $k$ are constants. The graph of $\ln y$ against $x^2$ is a straight line passing through the points (0.64, 0.76) and (1.69, 0.32), as shown in the diagram. Find the values of $A$ and $k$ correct to 2 decimal places. \[5\]

4 The polynomial $ax^3 - 20x^2 + x + 3$, where $a$ is a constant, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$.

(i) Find the value of $a$. \[3\]

(ii) When $a$ has this value, factorise $p(x)$ completely. \[3\]

5 The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where $a$ is a positive constant. The maximum point on the curve is $M$. Find the $x$-coordinate of $M$ in terms of $a$. \[6\]
6 (i) By differentiating \( \frac{1}{\cos x} \), show that the derivative of \( \sec x \) is \( \sec x \tan x \). Hence show that if \( y = \ln(\sec x + \tan x) \) then \( \frac{dy}{dx} = \sec x \). \[4\]

(ii) Using the substitution \( x = (\sqrt{3}) \tan \theta \), find the exact value of
\[
\int_{1}^{3} \frac{1}{\sqrt{3 + x^2}} \, dx,
\]
expressing your answer as a single logarithm. \[4\]

7 (i) By first expanding \( \cos(x + 45^\circ) \), express \( \cos(x + 45^\circ) - (\sqrt{2}) \sin x \) in the form \( R \cos(x + \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). Give the value of \( R \) correct to 4 significant figures and the value of \( \alpha \) correct to 2 decimal places. \[5\]

(ii) Hence solve the equation
\[
\cos(x + 45^\circ) - (\sqrt{2}) \sin x = 2,
\]
for \( 0^\circ < x < 360^\circ \). \[4\]

8 (i) Express \( \frac{1}{x^2(2x + 1)} \) in the form \( \frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x + 1} \). \[4\]

(ii) The variables \( x \) and \( y \) satisfy the differential equation
\[
y = x^2(2x + 1) \frac{dy}{dx},
\]
and \( y = 1 \) when \( x = 1 \). Solve the differential equation and find the exact value of \( y \) when \( x = 2 \). Give your value of \( y \) in a form not involving logarithms. \[7\]

9 (a) The complex number \( w \) is such that \( \text{Re} w > 0 \) and \( w + 3w^* = iw^2 \), where \( w^* \) denotes the complex conjugate of \( w \). Find \( w \), giving your answer in the form \( x + iy \), where \( x \) and \( y \) are real. \[5\]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers \( z \) which satisfy both the inequalities \( |z - 2i| \leq 2 \) and \( 0 \leq \arg(z + 2) \leq \frac{1}{4}\pi \). Calculate the greatest value of \( |z| \) for points in this region, giving your answer correct to 2 decimal places. \[6\]

10 The points \( A \) and \( B \) have position vectors \( 2i - 3j + 2k \) and \( 5i - 2j + k \) respectively. The plane \( p \) has equation \( x + y = 5 \).

(i) Find the position vector of the point of intersection of the line through \( A \) and \( B \) and the plane \( p \). \[4\]

(ii) A second plane \( q \) has an equation of the form \( x + by + cz = d \), where \( b \), \( c \) and \( d \) are constants. The plane \( q \) contains the line \( AB \), and the acute angle between the planes \( p \) and \( q \) is \( 60^\circ \). Find the equation of \( q \). \[7\]