1 Solve the equation \(|2^x - 7| = 1\), giving answers correct to 2 decimal places where appropriate. [5]

2 Solve the equation \(\ln(3 - 2x) - 2 \ln x = \ln 5\). [5]

3 (i) Show that \(12 \sin^2 x \cos^2 x = \frac{3}{2}(1 - \cos 4x)\). [3]

(ii) Hence show that \(\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} 12 \sin^2 x \cos^2 x \, dx = \frac{\pi}{8} + \frac{3\sqrt{3}}{16}\). [3]

4 The polynomial \(ax^3 - 5x^2 + bx + 9\), where \(a\) and \(b\) are constants, is denoted by \(p(x)\). It is given that \((2x + 3)\) is a factor of \(p(x)\), and that when \(p(x)\) is divided by \((x + 1)\) the remainder is 8.

(i) Find the values of \(a\) and \(b\). [5]

(ii) When \(a\) and \(b\) have these values, factorise \(p(x)\) completely. [3]

5 The parametric equations of a curve are \(x = e^{2t}, \; y = 4te^t\).

(i) Show that \(\frac{dy}{dx} = \frac{2(t + 1)}{e^t}\). [4]

(ii) Find the equation of the normal to the curve at the point where \(t = 0\). [4]

6 (i) By sketching a suitable pair of graphs, show that the equation \(\cot x = 4x - 2\), where \(x\) is in radians, has only one root for \(0 \leq x \leq \frac{1}{2}\pi\). [2]

(ii) Verify by calculation that this root lies between \(x = 0.7\) and \(x = 0.9\). [2]

(iii) Show that this root also satisfies the equation \(x = \frac{1 + 2 \tan x}{4 \tan x}\). [1]

(iv) Use the iterative formula \(x_{n+1} = \frac{1 + 2 \tan x_n}{4 \tan x_n}\) to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]
7 (i) Express \( 5 \sin 2\theta + 2 \cos 2\theta \) in the form \( R \sin(2\theta + \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), giving the exact value of \( R \) and the value of \( \alpha \) correct to 2 decimal places. \[3\]

Hence

(ii) solve the equation

\[ 5 \sin 2\theta + 2 \cos 2\theta = 4, \]

giving all solutions in the interval \( 0^\circ \leq \theta \leq 360^\circ \), \[5\]

(iii) determine the least value of \( \frac{1}{(10 \sin 2\theta + 4 \cos 2\theta)^2} \) as \( \theta \) varies. \[2\]