1 Solve the equation $|x^3 - 14| = 13$, showing all your working. \[4\]

2 The variables $x$ and $y$ satisfy the equation $y = A(b^x)$, where $A$ and $b$ are constants. The graph of $\ln y$ against $x$ is a straight line passing through the points $(0, 2.14)$ and $(5, 4.49)$, as shown in the diagram. Find the values of $A$ and $b$, correct to 1 decimal place. \[5\]

3 The polynomial $p(x)$ is defined by

$$p(x) = ax^3 - 3x^2 - 5x + a + 4,$$

where $a$ is a constant.

(i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of $a$. \[2\]

(ii) When $a$ has this value,

(a) factorise $p(x)$ completely, \[3\]

(b) find the remainder when $p(x)$ is divided by $(x + 1)$. \[2\]

4 (i) Given that $35 + \sec^2 \theta = 12 \tan \theta$, find the value of $\tan \theta$. \[3\]

(ii) Hence, showing the use of an appropriate formula in each case, find the exact value of

(a) $\tan(\theta - 45^\circ)$, \[2\]

(b) $\tan 2\theta$. \[2\]
The diagram shows the curve $y = 4e^{\frac{1}{2}x} - 6x + 3$ and its minimum point $M$.

(i) Show that the $x$-coordinate of $M$ can be written in the form $\ln a$, where the value of $a$ is to be stated. [5]

(ii) Find the exact value of the area of the region enclosed by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. [4]

6 A curve has parametric equations

$$x = \frac{1}{(2t + 1)^2}, \quad y = \sqrt{t + 2}.$$ 

The point $P$ on the curve has parameter $p$ and it is given that the gradient of the curve at $P$ is $-1$.

(i) Show that $p = (p + 2)^{\frac{1}{2}} - \frac{3}{2}$. [6]

(ii) Use an iterative process based on the equation in part (i) to find the value of $p$ correct to 3 decimal places. Use a starting value of 0.7 and show the result of each iteration to 5 decimal places. [3]

7 (i) Show that $(2 \sin x + \cos x)^2$ can be written in the form $\frac{5}{2} + 2 \sin 2x - \frac{3}{2} \cos 2x$. [5]

(ii) Hence find the exact value of $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (2 \sin x + \cos x)^2 \, dx$. [4]