1 Solve the inequality $|x + 3a| > 2|x - 2a|$, where $a$ is a positive constant. [4]

2 Solve the equation

$$\sin \theta = 2 \cos 2\theta + 1,$$

giving all solutions in the interval $0^\circ \leq \theta \leq 360^\circ$. [6]

3 The variables $x$ and $y$ satisfy the equation $x^n y = C$, where $n$ and $C$ are constants. When $x = 1.10$, $y = 5.20$, and when $x = 3.20$, $y = 1.05$.

(i) Find the values of $n$ and $C$. [5]

(ii) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [1]

4 (i) Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\frac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x.$$ [3]

(ii) Hence show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 3x \sin x \, dx = \frac{1}{8}\sqrt{3}.$$ [3]

5 Given that $y = 0$ when $x = 1$, solve the differential equation

$$xy \frac{dy}{dx} = y^2 + 4,$$

obtaining an expression for $y^2$ in terms of $x$. [6]
The diagram shows a semicircle $ACB$ with centre $O$ and radius $r$. The angle $BOC$ is $x$ radians. The area of the shaded segment is a quarter of the area of the semicircle.

(i) Show that $x$ satisfies the equation

$$x = \frac{3}{4} \pi - \sin x.$$  

(ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5.

(iii) Use the iterative formula

$$x_{n+1} = \frac{3}{4} \pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

7 The complex number $2 + 2i$ is denoted by $u$.

(i) Find the modulus and argument of $u$.

(ii) Sketch an Argand diagram showing the points representing the complex numbers $1$, $i$ and $u$. Shade the region whose points represent the complex numbers $z$ which satisfy both the inequalities $|z - 1| \leq |z - i|$ and $|z - u| \leq 1$.

(iii) Using your diagram, calculate the value of $|z|$ for the point in this region for which $\arg z$ is least.

8 (i) Express $\frac{2}{(x + 1)(x + 3)}$ in partial fractions.

(ii) Using your answer to part (i), show that

$$\left( \frac{2}{(x + 1)(x + 3)} \right)^2 = \frac{1}{(x + 1)^2} - \frac{1}{x + 1} + \frac{1}{x + 3} + \frac{1}{(x + 3)^2}.$$  

(iii) Hence show that

$$\int_0^1 \frac{4}{(x + 1)^2(x + 3)^2} \, dx = \frac{7}{12} - \ln \frac{3}{2}.$$
The diagram shows the curve \( y = \sqrt{\frac{1-x}{1+x}} \).

(i) By first differentiating \( \frac{1-x}{1-x} \), obtain an expression for \( \frac{dy}{dx} \) in terms of \( x \). Hence show that the gradient of the normal to the curve at the point \((x, y)\) is \((1 + x) \sqrt{1 - x^2}\). [5]

(ii) The gradient of the normal to the curve has its maximum value at the point \( P \) shown in the diagram. Find, by differentiation, the \( x \)-coordinate of \( P \). [4]

10 The lines \( l \) and \( m \) have vector equations

\[
\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})
\]

respectively.

(i) Show that \( l \) and \( m \) intersect. [4]

(ii) Calculate the acute angle between the lines. [3]

(iii) Find the equation of the plane containing \( l \) and \( m \), giving your answer in the form \( ax + by + cz = d \). [5]