This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2009 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Mark Scheme Notes

Marks are of the following three types:

M  Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A  Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B  Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking \( g \) equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO  Correct Working Only – often written by a ‘fortuitous’ answer
ISW  Ignore Subsequent Working
MR   Misread
PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR −1 A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR −2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA −1 This is deducted from A or B marks in the case of premature approximation. The PA −1 penalty is usually discussed at the meeting.
1. State or imply \(2 + e^{-x} = e^2\) \(\rightarrow B1\)
   Carry out method for finding \(x\) from \(e^{x} = k\), where \(k > 0\), following sound ln or exp work \(\rightarrow M1\)
   Obtain \(x = \ln(e^2 - 2)\), or equivalent expression for \(x\) \(\rightarrow A1\)
   Obtain answer \(x = -1.68\) \(\rightarrow A1\)
   [The answer must be given to 2 decimal places]
   [SR: the M1 is available for attempts starting with \(2 + e^{-x} = 10^2\)]

2. (i) State or imply 3 of the 4 ordinates 1, 1.069389..., 1.290994..., 1.732050...
   Use correct formula, or equivalent, with \(h = \frac{\pi}{12}\) and four ordinates \(\rightarrow M1\)
   Obtain answer 0.98 with no errors seen \(\rightarrow A1\)
   [Accept \(h = 0.26\) but not \(h = 15\) when awarding the M1]
   [SR: if only \(\frac{\pi}{3}\) and/or \(\frac{\pi}{\sqrt{3}}\) are given, and decimals are not seen, the B1 is available]
   [SR: solutions with 2 or 4 intervals can score only the M1 for a correct expression]
   \(\rightarrow B1\)

(ii) Justify statement that the second estimate would be less than \(E\) \(\rightarrow B1\)

3. (i) Use \(\cot A = 1/\tan A\) or \(\cos A/\sin A\) and/or \(\cosec A = 1/\sin A\) on at least two terms \(\rightarrow M1\)
   Use a correct double angle formula or the \(\sin(A - B)\) formula at least once \(\rightarrow M1\)
   Obtain given result \(\rightarrow A1\)
   \(\rightarrow B1\)

(ii) Solve \(\cot \theta = 2\) for \(\theta\) and obtain answer 26.6° \(\rightarrow B1\)
   Obtain answer 206.6° and no others in the given range \(\rightarrow B1\)
   [Ignore answers outside the given range. Treat answers given in radians as a misread]

4. (i) Compare signs of \(x^3 - 2x - 2\) when \(x = 1\) and \(x = 2\), or equivalent \(\rightarrow M1\)
   Complete the argument with correct calculations \(\rightarrow A1\)
   \(\rightarrow B1\)

(ii) State or imply the equation \(x = (2x^3 + 2) / (3x^2 - 2)\) \(\rightarrow B1\)
   Rearrange this in the form \(x^3 - 2x - 2 = 0\), or work \(vice versa\) \(\rightarrow B1\)

(iii) Use the iterative formula correctly at least once with \(x_n > 0\)
   Obtain final answer 1.77 \(\rightarrow A1\)
   Show sufficient iterations to 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change
   In the interval (1.765, 1.775) \(\rightarrow A1\)

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5 (i) State correct first two terms of the expansion of \((1 + ax)^{\frac{3}{2}}\), i.e. \(1 + \frac{3}{2}ax\)  

Form an expression for the coefficient of \(x\) in the expansion of \((1 + 2x)(1 + ax)^{\frac{3}{2}}\)  
and equate it to zero  
Obtain \(a = -3\)  

(ii) Obtain correct unsimplified terms in \(x^2\) and \(x^3\) in the expansion of \((1 - 3x)^{\frac{3}{2}}\)  

or \((1 + ax)^{\frac{3}{2}}\)  
Carry out multiplication by \(1 + 2x\) obtaining two terms in \(x^3\)  
Obtain final answer \(-\frac{10}{7}x^3\), or equivalent  

[Symbolic binomial coefficients, e.g. \(\binom{\frac{2}{3}}{1}\), are not acceptable for the B marks in (i) or (ii)]

6 (i) EITHER State \(\frac{dx}{dt} = -3a\cos^2 t\sin t\) or \(\frac{dy}{dt} = 3a\sin^2 t\cos t\), or equivalent  

Use \(\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}\)  

OR State \(\frac{2}{7}x^{-\frac{1}{2}}dx\) or \(\frac{2}{7}y^{-\frac{1}{2}}dy\) as differentials of \(x^{\frac{2}{7}}\) or \(y^{\frac{2}{7}}\) respectively,  
or equivalent  

Obtain \(\frac{dy}{dx}\) in terms of \(t\), having taken the differential of a constant to be zero  

Obtain \(\frac{dy}{dx}\) in any correct form  

(ii) Form the equation of the tangent  
Obtain the equation in any correct form  
Obtain the given answer  

(iii) State the \(x\)-coordinate of \(X\) or the \(y\)-coordinate of \(Y\) in any correct form  
Obtain the given answer with no errors seen
7  (i) Use quadratic formula, or completing the square, or the substitution \( z = x + iy \) to find a root, using \( i^2 = -1 \) M1
   Obtain a root, e.g. \( 1 - \sqrt{3}i \) A1
   Obtain the other root, e.g. \( -1 - \sqrt{3}i \) A1 3

(ii) Represent both roots on an Argand diagram in relatively correct positions B1 \( \sqrt{ } \) 1

(iii) State modulus of both roots is 2 B1 \( \sqrt{ } \)
   State argument of \( 1 - \sqrt{3}i \) is \(-60^\circ \) (or \(300^\circ, -\frac{\pi}{3}, -\frac{5\pi}{3} \)) B1 \( \sqrt{ } \)
   State argument of \( -1 - \sqrt{3}i \) is \(-120^\circ \) (or \(240^\circ, -\frac{2\pi}{3}, -\frac{4\pi}{3} \)) B1 \( \sqrt{ } \) 3

(iv) Give a complete justification of the statement B1 \( \sqrt{ } \)
   [The A marks in (i) are for the final versions of the roots. Allow \((\pm 2 - 2\sqrt{3}i)/2 \) as final answer. The remaining marks are only available for roots such that \(xy \neq 0\).] [Treat answers to (iii) in polar form as a misread]

8  (i) State or imply the form \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{10-x} \) \( \sqrt{ } \)
   Use any relevant method to determine a constant M1
   Obtain one of the values \( A = 1, B = 10, C = 1 \) A1
   Obtain the remaining two values A1 4
   [The form \( \frac{Dx + E}{x^2} + \frac{C}{10-x} \) is acceptable and leads to \( D = 1, E = 10, C = 1 \)]

(ii) Separate variables and attempt integration of both sides M1
   Obtain terms \( \ln x, -10/x, -\ln (10 - x) \), or equivalent A1 \( \sqrt{ } \) + A1 \( \sqrt{ } \) + A1 \( \sqrt{ } \\
   Evaluate a constant or use limits \( x = 1, t = 0 \) with a solution containing 3 of the terms \( k\ln x, l/x, m\ln (10-x) \) and \( t \), or equivalent M1
   Obtain any correct expression for \( t \), e.g. \( t = \ln \left( \frac{9x}{10-x} \right) - \frac{10}{x} + 10 \) A1 6
   [A separation of the form \( \frac{adx}{x^2(10-x)} = bdx \) is essential for the M1. The f.t. is on \( A, B, C \)]
   [If \( A \) or \( B \) (or \( D \) or \( E \)) omitted from the form of fractions, give B0M1A0A0 in (i); M1A1 \( \sqrt{ } \) A1 \( \sqrt{ } \) M1A0 in (ii)]
9 (i) EITHER Substitute coordinates of general point of \( l \) in equation of plane and equate constant terms, obtaining an equation in \( b \) and \( c \) M1 *
Obtain a correct equation, e.g. \( 8 + 2b - c = 1 \) A1
Equate the coefficient of \( t \) to zero, obtaining an equation in \( b \) and \( c \) M1 *
Obtain a correct equation, e.g. \( 4 - b - 2c = 0 \) A1

OR Substitute \((4, 2, -1)\) in the plane equation M1 *
Obtain a correct equation in \( b \) and \( c \), e.g. \( 2b - c = -7 \) A1
EITHER Find a second point on \( l \) and obtain an equation in \( b \) and \( c \) M1 *
Obtain a correct equation in \( b \) and \( c \), e.g. \( b + 2c = 4 \) A1
OR Calculate scalar product of a direction vector for \( l \) and a vector normal for the plane and equate to zero M1 *
Obtain a correct equation for \( b \) and \( c \) A1
Solve for \( b \) or for \( c \) M1 (dep*)
Obtain \( b = -2 \) and \( c = 3 \) A1

(ii) EITHER Find \( \overrightarrow{PQ} \) for a point \( Q \) on \( l \) with parameter \( t \), e.g. \( 4i - 5k + t(2i - j - 2k) \) B1
Calculate scalar product of \( \overrightarrow{PQ} \) and a direction vector for \( l \) and equate to zero M1
Solve and obtain \( t = -2 \) A1
Carry out a complete method for finding the length of \( \overrightarrow{PQ} \) M1
Obtain the given answer \( \sqrt{5} \) correctly A1

OR 1 Calling \((4, 2, -1)\) \( A \), state \( \overrightarrow{AP} \) (or \( \overrightarrow{PA} \)) in component form, e.g. \( 4i - 5k \) B1
Calculate vector product of \( \overrightarrow{AP} \) and a direction vector for \( l \), e.g. \( (4i - 5k) \times (2i - j - 2k) \) M1
Obtain correct answer, e.g. \(-5i - 2j - 4k \) A1
Divide modulus of the product by that of the direction vector M1
Obtain the given answer correctly A1

OR 2 State \( \overrightarrow{AP} \) (or \( \overrightarrow{PA} \)) in component form B1
Use a scalar product to find the projection of \( \overrightarrow{AP} \) (or \( \overrightarrow{PA} \)) on \( l \) M1
Obtain correct answer in any form, e.g. \( \frac{18}{\sqrt{9}} \) A1
Use Pythagoras to find the perpendicular M1
Obtain the given answer correctly A1

OR 3 State \( \overrightarrow{AP} \) (or \( \overrightarrow{PA} \)) in component form B1
Use a scalar product to find the cosine of \( PAQ \) M1
Obtain correct answer in any form, e.g. \( \frac{18}{\sqrt{41 \cdot 9}} \) A1
Use trig to find the perpendicular M1
Obtain the given answer correctly A1
### OR 4
State \( \overrightarrow{AP} \) (or \( \overrightarrow{PA} \)) in component form
Find a second point \( B \) on \( l \) and use the cosine rule in triangle \( APB \) to find the cosine of \( A, B \) or \( P \), or use a vector product to find the area of \( APB \)
Obtain correct answer in any form
Use trig or area formula to find the perpendicular
Obtain the given answer correctly

### OR 5
Find \( \overrightarrow{PQ} \) for a point \( Q \) on \( l \) with parameter \( t \), e.g. \( 4i - 5k + t(2i - j - 2k) \)
Use correct method to express \( PQ^2 \) (or \( PQ \)) in terms of \( t \)
Obtain a correct expression in any form, e.g. \( (4 + 2t)^2 + (-1)^2 + (-5 - 2t)^2 \)
Carry out a complete method for finding its minimum
Obtain the given answer correctly

10 (i) EITHER
Use product and chain rule
Obtain correct derivative in any form

OR
Square and differentiate LHS by chain rule and RHS by product rule or as powers
Obtain correct result in any form

Set \( \frac{dy}{dx} \) equal to zero and make reasonable attempt to solve for \( x \neq 0 \)
Obtain answer \( x = \sqrt[3]{\frac{\alpha}{3}} \), or exact equivalent, correctly

(ii) State or imply \( dx = \cos \theta \, d\theta \) or \( \frac{dx}{d\theta} = \cos \theta \)
Substitute for \( x \) and \( dx \) throughout the integral \( \int ydx \)
Obtain the given form correctly with no errors seen

(iii) Attempt integration and reach indefinite integral of the form \( a\theta + b \sin 4\theta \), where \( ab \neq 0 \)
Obtain indefinite integral \( \frac{1}{8} \theta - \frac{1}{32} \sin 4\theta \), or equivalent
Substitute limits correctly
Obtain exact answer \( \frac{1}{16} \pi \)

[Working to carry out the change of limits is needed for the A mark in (ii) but, if omitted, can be earned retrospectively if it is seen in part (iii)]