CAMBRIDGE INTERNATIONAL EXAMINATIONS
GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the June 2005 question papers

9709 MATHEMATICS
8719 HIGHER MATHEMATICS

8719/07, 9709/07 - Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published Report on the Examination.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the Report on the Examination.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2005 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.
Grade thresholds for Syllabus 8719 and 9709 (Mathematics and Higher Mathematics) in the June 2005 examination.

<table>
<thead>
<tr>
<th>Component 7</th>
<th>maximum mark available</th>
<th>minimum mark required for grade:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>39</td>
</tr>
</tbody>
</table>

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.
Mark Scheme Notes

- Marks are of the following three types:

  **M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  **A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  **B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2,1, 0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking \( g \) equal to 9.8 or 9.81 instead of 10.
The following abbreviations may be used in a mark scheme or used on the scripts:

**AEF** Any Equivalent Form (of answer is equally acceptable)

**AG** Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

**BOD** Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

**CAO** Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

**CWO** Correct Working Only - often written by a ‘fortuitous’ answer

**ISW** Ignore Subsequent Working

**MR** Misread

**PA** Premature Approximation (resulting in basically correct work that is insufficiently accurate)

**SOS** See Other Solution (the candidate makes a better attempt at the same question)

**SR** Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### Penalties

**MR -1** A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

**PA -1** This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.
| 1 | $55 = 70a + b$  
$6.96 = 8.7a$ or $6.96^2 = 8.7^2a^2$  
$a = 0.8$  
$b = -1$ | M1 | For an equation relating to the means  
For an equation relating to the variance or $sd$, only $a$ in it  
For correct $a$  
For correct $b$ | M1 A1 A1 (4) |
|---|---|---|---|---|
| 2 | (i) Put names in a hat and draw out, or assign a number to each person in year and generate 7 random numbers by calculator.  
(ii) est pop mean $116.5/7$ (= 16.6)  
est pop var = 27.1  
(iii) more  
(iv) (pocket money of) all pupils in Jenny’s year at school | B1 (1) | Or any equivalent method, could use systematic sampling |  
B1 | For using a correct formula (can be implied)  
A1 (3) | For correct answer  
B1 (1) | Need to see all of this |
| 3 | (i) $(0.1993 + 0.2887)/2$ (= 0.244)  
$= 61/n$  
$n = 250$  
(ii) $0.0447 = z \times \sqrt{0.244(1-0.244)/250}$  
(or equiv. equ. leading to this)  
z = 1.646  
90% confidence interval | B1 M1 A1 (3) | For correct mid-point  
For equating their mid-point with $61/n$  
For correct answer | M1 | For equating half-width with $z \times \sqrt{pq/n}$ or equiv.  
A1 | For solving for $z$ from a reasonable looking equation  
| A1ft (4) | For obtaining $z = 1.64$ or 1.65  
|  
| 4 | (i) $H_0$: $\mu = 21.2$  
$H_1$: $\mu \neq 21.2$  
Test statistic $z = \frac{19.4 - 21.2}{(7.3/\sqrt{90})}$  
$= -2.34$  
CV $z = \pm 1.96$  
In CR, reject $H_0$. Sig evidence to say not the same author  
or $\Phi (-2.339) = 1 - 0.9903$  
$= 0.0097/0.0096$  
Compare with 0.025  
say sig evidence to say not the same sentence length or author  
or $x = 21.2 \pm 1.96x(7.3/\sqrt{90})$  
$= 19.7(22.7)$  
Compare with 19.4 etc.  
(ii) Say it is not the same sentence length or author when it is  
P (Type I error) = 5% | B1 | For $H_0$ and $H_1$ correct must be $\neq$  
For standardising must have $\sqrt{90}$  
A1 | For correct $z$ accept +/-  
M1 | For correct comparison with correct critical value, ft from their $H_1$  
A1ft (5) | For correct conclusion ft on their $z$ and their CV  
| M1 | For correct $\Phi$, and correct comparison (consistent with $H_1$)  
A1ft | For correct conclusion ft on their $\Phi$ and 0.025  
| M1 A1 | For expression for $x$ with correct (consistent) $z$  
M1 A1ft | For correct comparison and conclusion(ft)  
B1 | For correct statement  
B1 (2) | For correct answer |
5  (i) $T \sim N(1.54 \times 4, 0.05^2 \times 4)$ 

$\left[ \sim N(6.16, 0.01) \right]$ 

$P(T > 5.95) = 1 - \Phi \left( \frac{(5.95-6.16)}{\sqrt{0.01}} \right)$ 

$= \Phi (2.1)$ 

$= 0.982$

(ii) $Av \sim N(6.16, 0.01/20)$ 

$P(Av > 6.2) = 1 - \Phi \left( \frac{(6.2 - 6.16)}{\sqrt{(0.01/20)}} \right)$ 

$= 1 - \Phi (1.789)$ 

$= 1 - 0.9633$ 

$= 0.0367$ or $0.0368$

or $Tot \sim N(123.2, 0.2)$ 

$P(Tot > 124) = 1 - \Phi \left( \frac{(123.2 - 124)}{\sqrt{0.2}} \right)$ etc.

| B1 | For mult mean and variance by 4 |
| M1 | For standardising must have $\sqrt{}$ |
| M1 | For correct area i.e. > 0.5 |
| A1 (4) | For correct answer |
| B1ft | For dividing their variance by 20 |
| M1 | For standardising (must use consistent values) |

6  (i) (a) East $P(\geq 1) = 1 - e^{-2} = 0.8647$ 

West $P(\geq 1) = 1 - e^{-1.25}$ 

$= 0.7135$

$P(\text{Both}) = 0.8647 \times 0.7135$ 

$= 0.617$

(b) $P(\text{total} \geq 2) = 1 - e^{-13/4}(1+13/4)$ 

$= 0.835$

or $P(\text{total} \geq 2) = P(2) + P(3) +...P(13)$ etc.

| B1 | Correct mean of 2 or 1.25 used in Poisson expression |
| M1 | One Poisson expression $P(\geq 1)=1 - P(0)$ or $1 - P(0) - P(1)$ any mean. |
| M1 | For multiplying their 2 probs together |
| A1 (4) | For correct answer |
| M1 | For attempt at summing their means and for 1 - their $P(0, 1)$ or 1 - their $P(0,1,2)$ or $1 - P(0E,0W) - P(1E,0W) - P(0E,1W)$ or equiv. expression incl.2 |
| A1 (2) | For correct answer |

(ii) $T \sim N(156, 156)$ 

$P(>175) = 1 - \Phi \left( \frac{(175.5 - 156)}{\sqrt{156}} \right)$ 

$= 1 - \Phi (1.5612)$ 

$= 1 - 0.9407$ 

$= 0.0593/0.0592$

| B1 | For correct mean and variance |
| M1 | For standardising, with or without cc or sq rt |
| A1 (3) | For correct answer |
7 (i) \[ \int_{0}^{24} \frac{(x-18)^2}{k} \, dx = 1 \]
\[ \left[ \frac{(x-18)^3}{3k} \right]_0^{24} = 1 \]
\[ \frac{2016}{k} = 1 \implies k = 2016 \text{ AG} \]

(ii) \( p(x < 2) = \int_{0}^{2} \frac{(x-18)^2}{2016} \, dx \)
\[ = \left[ \frac{(x-18)^3}{3 \times 2016} \right]_0^{2} \]
\[ = \frac{(-16)^3 - (-18)^3}{6048} \]
\[ = 0.2870(31/108) \]
Number of days = \( 0.287 \times 365 = 104 \text{ or } 105 \)

(iii) \( \text{mean} = \int_{0}^{24} \frac{x(x-18)^2}{k} \, dx \)
\[ = \frac{1}{k} \left[ \frac{x^4}{4} - \frac{36x^3}{3} + \frac{324x^2}{2} \right]_0^{24} \]
\[ = 5.14 \left( \frac{5}{7} \right) \]

\[ M1 \quad \text{For equating to 1 and attempting to integrate} \]
\[ A1 \quad \text{For correct integration with correct limits seen} \]
\[ A1 (3) \quad \text{For given answer legit obtained} \]

\[ M1 \quad \text{For integration attempt between 0 and 2 (condone missing } k \text{)} \]
\[ A1 \quad \text{For correct answer} \]
\[ B1ft (3) \quad \text{For multiplying their prob by 365} \]

\[ M1 \quad \text{For attempting to integrate } xf(x) \]
(\text{condone missing } k) \]
\[ A1 \quad \text{For one correct integrated term with correct limits(\text{condone missing } k)} \]
\[ \text{or For integration by parts correct first stage answer with limits seen(\text{condone missing } k)} \]
\[ A1 \quad \text{For fully correct integrated expression with limits(\text{condone missing } k)} \]
\[ A1 (4) \quad \text{For correct answer} \]