These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners’ meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published Report on the Examination.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the Report on the Examination.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the June 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.
**Grade thresholds** taken for Syllabus 9709 (Mathematics) in the June 2004 examination.

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The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.
Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0. B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.
• The following abbreviations may be used in a mark scheme or used on the scripts:

AEF  Any Equivalent Form (of answer is equally acceptable)
AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD  Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO  Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO  Correct Working Only – often written by a ‘fortuitous’ answer
ISW  Ignore Subsequent Working
MR   Misread
PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS  See Other Solution (the candidate makes a better attempt at the same question)
SR   Special Ruling (detailling the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

• MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

• PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.
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<td>MATHEMATICS</td>
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<td>Paper 1 (Pure 1)</td>
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1. (i) \( a/(1-r) = 256 \) and \( a = 64 \)
   \[ r = \frac{1}{4} \]

   (ii) \( S_{10} = 64(1-0.75^{10}) \times (1-0.75) \)
   \[ S_{10} = 242 \]

2. \[ \int_{0}^{1} \sqrt{3x+1} \, dx = \left(3x+1\right)^{1.5} \div 1.5 \]
   then \( 3 \)
   \[ \left[ \right] \text{at } 1 - \left[ \right] \text{at } 0 \]
   \[ 16/9 - 2/9 = 14/9 \text{ or } 1.56 \]

3. (i) \( \sin^2 \theta + 3 \sin \theta \cos \theta = 4 \cos^2 \theta \)
   divides by \( \cos^2 \theta \)
   \[ \tan^2 \theta + 3 \tan \theta = 4 \]

   (ii) Solution \( \tan \theta = 1 \) or \( \tan \theta = -4 \)
   \[ \theta = 45^\circ \text{ or } 104.0^\circ \]

4. (i) Coeff of \( x^3 \) = \( 6C3 \times 2^3 \)
   \[ = 160 \]

   (ii) Term in \( x^2 \) = \( 6C2 \times 2^2 = 60 \)
   reqd coeff = \( 1 \times (i) - 3 \times 60 \)
   \[ \rightarrow -20 \]

5. (i) Area of sector = \( \frac{1}{2} \times 6^2 \times 0.8 \) (14.4)
   Area of triangle = \( \frac{1}{2} \times 10 \times \sin0.8 \) (35.9)
   → Shaded area = 21.5

   (ii) Arc length = \( 6 \times 0.8 \) (4.8)
   CD (by cos rule) or \( 2 \times 10 \sin0.4 \) (7.8)
   → Perimeter = \( 8 + 4.8 + 7.8 = 20.6 \)
<table>
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<th>Question</th>
<th>Mark Scheme</th>
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<tr>
<td>6. (i) eliminates x (or y) completely → ( x^2 + x - 6 = 0 ) or ( y^2 - 17y + 66 = 0 )</td>
<td>M1 Needs x or y removed completely A1 Correct only (no need for = 0) DM1 Equation must = 0. A1 Everything ok. [4]</td>
</tr>
<tr>
<td>(ii) Midpoint = ((-\frac{1}{2}, 8\frac{1}{2})) Gradient of line = (-1) Gradient of perpendicular = 1</td>
<td>B1 √ For his two points in (i) Use of y-step x-step (beware fortuitous) M1 Use of (m_1m_2 = -1) M1 A1 Any form – needs the M marks. [4]</td>
</tr>
<tr>
<td>7. (i) Differentiate ( y = \frac{18}{x} ) →</td>
<td>M1 Any attempt at differentiation A1 For (-\frac{1}{2}) DM1 Use of (m_1m_2 = -1) DM1 Correct method for eqn of line M1 A1 Ans given – beware fortuitous answers. [5]</td>
</tr>
<tr>
<td>(ii) Vol = ( \pi \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{324}{x^2} dx = \pi \left[ -\frac{324}{x} \right] ).</td>
<td>M1 A1 Use of ( \int y^2 dx ) for M. correct (needs ( \pi )) for A DM1 Use of 6 and 4.5 A1 Beware fortuitous answers (ans given) [4]</td>
</tr>
<tr>
<td>8. (i) ( 2h + 2r + \pi r = 8 )</td>
<td>M1 Reasonable attempt at linking 4 lengths + correct formula for ( \frac{1}{2}C ) or C. A1 Co in any form with h subject. [2]</td>
</tr>
<tr>
<td>→ ( h = 4 - r - \frac{1}{2} \pi r )</td>
<td>[2]</td>
</tr>
<tr>
<td>(ii) ( A = 2rh + \frac{1}{2} \pi r^2 \rightarrow A = r(8 - 2r - \pi r) + \frac{1}{2} \pi r^2 ) ( \rightarrow A = 8r - 2r^2 - \frac{1}{2} \pi r^2 )</td>
<td>M1 Adds rectangle + ( \frac{1}{2} )xcircle (eqn on own ok) A1 Co beware fortuitous answers (ans given) [2]</td>
</tr>
<tr>
<td>(iii) ( \frac{dA}{dr} = 8 - 4r - \pi r ) = 0 when ( r = 1.12 ) (or ( 8/(4+ \pi) ))</td>
<td>M1 A1 Knowing to differentiate + some attempt DM1 A1 Setting his ( dA/dr ) to 0. Decimal or exact ok. [4]</td>
</tr>
<tr>
<td>(iv) ( \frac{d^2A}{dr^2} = -4 - \pi ) This is negative → Maximum</td>
<td>M1 Looks at ( 2^{nd} ) differential or other valid complete method. A1 Correct deduction but needs ( d^2A/dr^2 ) correct. [2]</td>
</tr>
</tbody>
</table>
9. \( \overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix}, \overrightarrow{OD} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix} \)

(i) \( \overrightarrow{AB} = \mathbf{b} - \mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k} \)

Unit vector = \( (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) \sqrt{(2^2 + 4^2 + 4^2)} \)

= \( \pm (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) \) 6

(ii) \( \overrightarrow{OA} \cdot \overrightarrow{OC} = 4 + 6 - p \)

\( = 0 \) for 90°

\( \rightarrow p = 10 \)

(iii) \( (-2)^2 + 3^2 + (q+1)^2 = 7^2 \)

\( \rightarrow (q+1)^2 = 36 \) or \( q^2 + 2q = 35 \)

\( q = 5 \) and \( q = -7 \)

Condone notation throughout.

Allow column vectors or \( i,j,k \) throughout

Use of \( \mathbf{b} - \mathbf{a} \), rather than \( \mathbf{b} + \mathbf{a} \) or \( \mathbf{a} - \mathbf{b} \)

Dividing by the modulus of "his" \( \overrightarrow{AB} \)

Co (allow – for candidates using \( \mathbf{a} - \mathbf{b} \))

Use of \( x_1x_2 + y_1y_2 + z_1z_2 \)

Setting to 0 + attempt to solve co

Correct method for length with \( \pm \mathbf{d} - \mathbf{a}, \mathbf{d} + \mathbf{a} \)

Correct quadratic equation

Correct method of solution. Both correct. Or \( \text{B1 for each if } (q+1)^2=36, q=5 \) only.

10. \( f: x \mapsto x^2 - 2x, \quad g: x \mapsto 2x + 3 \)

(i) \( x^2 - 2x - 15 = 0 \)

End-points \(-3\) and \(5\)

\( \rightarrow x < -3 \) and \( x > 5 \)

(ii) Uses \( \frac{dy}{dx} = 2x - 2 = 0 \) or \( (x - 1)^2 - 1 \)

Minimum at \( x = 1 \) or correct form

Range of \( y \) is \( f(x) \geq -1 \)

No inverse since not \( 1:1 \) (or equivalent)

(iii) \( gf(x) = 2(x^2 - 2x) + 3 \) (or \( (2x^2 - 4x + 3) \))

\( b^2 - 4ac = 16 - 24 = -8 \rightarrow -\text{ve} \)

\( \rightarrow \) No real solutions.

\[ \text{[or } gf(x) = 0 \rightarrow f(x) = -3/2. \text{ Impose from (ii)]} \]

(iv) \( y = 2x + 3 \) correct line on diagram

Either inverse as mirror image in \( y=x \)

\( \text{or } y = g^{-1}(x) = \frac{1}{2} (x-3) \) drawn

Equation set to 0 and solved.

Correct end-points, however used

Co-inequalities – not \( \leq \) or \( \geq \)

Any valid complete method for \( x \) value

Correct only

Correct for his value of "\( x \)" – must be \( \geq \)

Any valid statement.

Must be \( g \) not \( fg \) – for unsimplified ans.

Used on quadratic=0, even if \( fg \) used.

Must be using \( gf \) and correct assumption and statement needed.

3 things needed – \( \text{B1 if one missing.} \)

- \( g \) correct,
- \( g^{-1} \) correct – not parallel to \( g \)
- \( y=x \) drawn or statement re symmetry

DM1 for quadratic equation. Equation must be set to 0.

Formula \( \rightarrow \) must be correct and correctly used – allow for numerical errors though in \( b^2 \) and \( -4ac \).

Factors \( \rightarrow \) attempt to find 2 brackets. Each bracket then solved to 0.
1. Use logarithms to linearise an equation
   \[ \frac{x}{y} = \frac{\ln 5}{\ln 2} \] or equivalent
   Obtain answer 2.32
   M1 A1 A1

2. (i) Use the given iterative formula correctly at least ONCE with \( x_1 = 3 \)
   Obtain final answer 3.142
   Show sufficient iterations to justify its accuracy to 3 d.p.
   M1 A1 A1

   (ii) State any suitable equation e.g. \( x = \frac{1}{5} \left( 4x + \frac{306}{x^4} \right) \)
   Derive the given answer \( \alpha \) or \( x \) = \( \sqrt{306} \)
   B1 B1

3. (i) Substitute \( x = 3 \) and equate to zero
   Obtain answer \( \alpha = -1 \)
   M1 A1

   (ii) At any stage, state that \( x = 3 \) is a solution
   EITHER: Attempt division by \((x−3)\) reaching a partial quotient of \(2x^2 + kx\)
   Obtain quadratic factor \(2x^2 + 5x + 2\)
   Obtain solutions \( x = -2 \) and \( x = -\frac{1}{2} \)
   A1 A1 A1
   OR: Obtain solution \( x = -2 \) by trial and error
   Obtain solution \( x = -\frac{1}{2} \) similarly
   B1 B2

4. (i) State answer \( R = 5 \)
   Use trigonometric formulae to find \( \alpha \)
   Obtain answer \( \alpha = 53.13^\circ \)
   B1 M1 A1

   (ii) Carry out, or indicate need for, calculation of \( \sin^{-1}(4.5/5) \)
   Obtain answer 11.0°
   A1✓
   Carry out correct method for the second root e.g. \( 180^\circ - 64.16^\circ - 53.13^\circ \)
   Obtain answer 62.7° and no others in the range
   A1✓ 4
   [Ignore answers outside the given range.]

   (iii) State least value is 2
   B1✓

5. (i) State derivative of the form \( (e^x \pm xe^x) \). Allow \( xe^x \pm e^x \) (via quotient rule)
   Obtain correct derivative of \( e^x - xe^x \)
   Equate derivative to zero and solve for \( x \)
   Obtain answer \( x = 1 \)
   M1 A1 M1 A1

   (ii) Show or imply correct ordinates \( 0, 0.367879..., 0.27067... \)
   Use correct formula, or equivalent, with \( h = 1 \) and three ordinates
   Obtain answer 0.50 with no errors seen
   B1 M1 A1

   (iii) Justify statement that the rule gives an under-estimate
   B1

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| 6 | (i) State that \( \frac{dx}{dt} = 2 + \frac{1}{t} \) or \( \frac{dy}{dt} = 1 - \frac{4}{t^2} \), or equivalent B1
   | Use \( \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} \) M1
   | Obtain the given answer A1 3 |
|   | (ii) Substitute \( t = 1 \) in \( \frac{dy}{dx} \) and both parametric equations M1
   | Obtain \( \frac{dy}{dx} = -1 \) and coordinates (2, 5) A1
   | State equation of tangent in any correct horizontal form e.g. \( x + y = 7 \) A1 3 |
|   | (iii) Equate \( \frac{dy}{dx} \) to zero and solve for \( t \) M1
   | Obtain answer \( t = 2 \) A1
   | Obtain answer \( y = 4 \) A1
   | Show by any method (but not via \( \frac{d}{dt}(y') \)) that this is a minimum point A1 4 |
| 7 | (i) Make relevant use of the cos(A + B) formula M1*
   | Make relevant use of cos2A and sin2A formulae M1*
   | Obtain a correct expression in terms of cosA and sinA A1
   | Use \( \sin^2 A = 1 - \cos^2 A \) to obtain an expression in terms of cosA M1(dep*)
   | Obtain given answer correctly A1 5 |
|   | (ii) Replace integrand by \( \frac{1}{4} \cos 3x + \frac{3}{4} \cos x \), or equivalent B1
   | Integrate, obtaining \( \frac{1}{12} \sin 3x + \frac{3}{4} \sin x \), or equivalent B1 + B1
   | Use limits correctly M1
<p>| Obtain given answer A1 5 |</p>
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<td>MATHEMATICS AND HIGHER MATHEMATICS</td>
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<td>Paper 3 (Pure 3)</td>
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1. Show correct sketch for \( 0 \leq x < \frac{1}{2}\pi \)  
Show correct sketch for \( \frac{1}{2}\pi < x < \frac{3}{2}\pi \) or \( \frac{3}{2}\pi < x \leq 2\pi \)  
Show completely correct sketch  
[SR: for a graph with \( y = 0 \) when \( x = 0, \pi, 2\pi \) but otherwise of correct shape, award B1.]

2. EITHER: State or imply non-modular inequality \((2x + 1)^2 < x^2\) or corresponding quadratic equation or pair of linear equations \((2x + 1) = \pm x\)  
Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations  
Obtain critical values \( x = -1 \) and \( x = -\frac{1}{3} \) only  
State answer \(-1 < x < -\frac{1}{3}\)  
OR: Obtain the critical value \( x = -1 \) from a graphical method, or by inspection, or by solving a linear inequality or equation  
Obtain the critical value \( x = -\frac{1}{3} \) (deduct B1 from B3 if extra values are obtained)  
State answer \(-1 < x < -\frac{1}{3}\)  
[Condone \( \leq \) for \(<\); accept \(-0.33\) for \(-\frac{1}{3}\).]

3. EITHER: State \( 6y \frac{dy}{dx} \) as the derivative of \( 3y^2 \)  
State \( \pm 4x \frac{dy}{dx} \pm 4y \) as the derivative of \( -4xy \)  
Equate attempted derivative of LHS to zero and solve for \( \frac{dy}{dx} \)  
Obtain answer 2  
[The M1 is conditional on at least one of the B marks being obtained. Allow any combination of signs for the second B1.]

OR: Obtain a correct expression for \( y \) in terms of \( x \)  
Differentiate using chain rule  
Obtain derivative in any correct form  
Substitute \( x = 2 \) and obtain answer 2 only  
[The M1 is conditional on a reasonable attempt at solving the quadratic in \( y \) being made.]
4 (i) State or imply \(2^{-x} = \frac{1}{y}\)  
Obtain 3-term quadratic e.g. \(y^2 - y - 1 = 0\) \(\text{B1} \quad \text{B1} \quad \text{2}\)

(ii) Solve a 3-term quadratic, obtaining 1 or 2 roots \(\text{M1}\)
Obtain answer \(y = (1 + \sqrt{5})/2\), or equivalent \(\text{A1}\)
Carry out correct method for solving an equation of the form \(2^x = a\), where \(a > 0\), reaching a ratio of logarithms \(\text{M1}\)
Obtain answer \(x = 0.694\) only \(\text{A1}\)

5 (i) Make relevant use of formula for \(\sin 2\theta\) or \(\cos 2\theta\) \(\text{M1}\)
Make relevant use of formula for \(\cos 4\theta\) \(\text{M1}\)
Complete proof of the given result \(\text{A1} \quad \text{3}\)

(ii) Integrate and obtain \(\frac{1}{3}(\theta - \frac{1}{4}\sin 4\theta)\) or equivalent \(\text{B1}\)
Use limits correctly with an integral of the form \(a\theta + b\sin 4\theta\), where \(ab \neq 0\) \(\text{M1}\)
Obtain answer \(\frac{1}{8}(\frac{1}{3}\pi + \frac{\sqrt{3}}{8})\), or exact equivalent \(\text{A1} \quad \text{3}\)

6 Separate variables and attempt to integrate \(\text{M1}\)
Obtain terms \(\frac{1}{3}\ln(y^3 + 1)\) and \(x\), or equivalent \(\text{A1} \quad \text{A1}\)
Evaluate a constant or use limits \(x = 0, y = 1\) with a solution containing terms \(k\ln(y^3 + 1)\) and \(x\), or equivalent \(\text{M1}\)
Obtain any correct form of solution e.g. \(\frac{1}{3}\ln(y^3 + 1) = x + \frac{1}{3}\ln 2\) \(\text{A1}\)
Rearrange and obtain \(y = (2e^{3x} - 1)^{\frac{1}{3}}\), or equivalent \(\text{A1} \quad \text{6}\)  
[f.t. is on \(k \neq 0\).]

7 (i) Evaluate cubic when \(x = -1\) and \(x = 0\) \(\text{M1}\)
Justify given statement correctly \(\text{A1} \quad \text{2}\)
[If calculations are not given but justification uses correct statements about signs, award B1.]

(ii) State \(x = \frac{2x^3 - 1}{3x^2 + 1}\), or equivalent \(\text{B1}\)
Rearrange this in the form \(x^3 + x + 1 = 0\) (or vice versa) \(\text{B1} \quad \text{2}\)
(iii) Use the iterative formula correctly at least once

Obtain final answer \(-0.68\) \(A1\)

Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval \((-0.685, -0.675)\) \(A1\)

8 (i) EITHER: Solve the quadratic and use \(\sqrt{-1} = i\) \(M1\)

Obtain roots \(\frac{1}{2} + i\frac{\sqrt{3}}{2}\) and \(\frac{1}{2} - i\frac{\sqrt{3}}{2}\) or equivalent \(A1\)

OR: Substitute \(x + iy\) and solve for \(x\) or \(y\) \(M1\)

Obtain correct roots \(A1\)

(ii) State that the modulus of each root is equal to 1 \(B1\)

State that the arguments are \(\frac{\pi}{3}\) and \(-\frac{\pi}{3}\) respectively \(B1 + B1\)

[Accept degrees and \(\frac{5}{3}\pi\) instead of \(-\frac{\pi}{3}\). Accept a modulus in the form \(\sqrt[\frac{p}{q}]{n}\) or \(\sqrt[n]{p}\), where \(p, q, n\) are integers. An answer which only gives roots in modulus-argument form earns \(B1\) for both the implied moduli and \(B1\) for both the implied arguments.]

(iii) EITHER: Verify \(z^3 = -1\) for each root \(B1 + B1\)

OR: State \(z^3 + 1 = (z + 1)(z^2 - z + 1)\) \(B1\)

Justify the given statement \(B1\)

OR: Obtain \(z^3 = z^2 - z\) \(B1\)

Justify the given statement \(B1\)

9 (i) State or imply \(f(x) = \frac{A}{x - 1} + \frac{B}{x - 2} + \frac{C}{x + 1}\) \(B1\)

EITHER: Use any relevant method to obtain a constant \(M1\)

Obtain one of the values: \(A = -1, B = 4\) and \(C = -2\) \(A1\)

Obtain the remaining two values \(A1\)

OR: Obtain one value by inspection \(B1\)

State a second value \(B1\)

State the third value \(B1\)

[Apply the same scheme to the form \(\frac{A}{x - 2} + \frac{Bx + C}{x^2 - 1}\) which has \(A = 4, B = -3\) and \(C = 1\).]
(ii) Use correct method to obtain the first two terms of the expansion of $(x - 1)^{-1}$ or $(x - 2)^{-1}$ or $(x + 1)^{-1}$  
M1
Obtain any correct unsimplified expansion of the partial fractions up to the terms in $x^3$
(deduct A1 for each incorrect expansion)
A1 $\sqrt{}$ + A1 $\sqrt{}$ + A1 $\sqrt{}$
Obtain the given answer correctly
A1

[Binomial coefficients involving $-1$, e.g. $\binom{-1}{1}$, are not sufficient for the M1 mark. The f.t. is on $A$, $B$, $C$.]

[Apply a similar scheme to the alternative form of fractions in (i), awarding M1*A1 $\sqrt{}$A1 $\sqrt{}$ for the expansions, M1(dep*) for multiplying by $Bx + C$, and A1 for obtaining the given answer correctly.]

[In the case of an attempt to expand $(x^2 + 7x - 6)(x - 1)^{-1}(x - 2)^{-1}(x + 1)^{-1}$, give M1A1A1A1 for the expansions and A1 for multiplying out and obtaining the given answer correctly.]

[Allow attempts to multiply out $(x - 1)(x - 2)(x + 1)(-3 + 2x - \frac{3}{2}x^2 + \frac{11}{4}x^3)$, giving B1 for reduction to a product of two expressions correct up to their terms in $x^3$, M1 for attempting to multiply out at least as far as terms in $x^2$, A1 for a correct expansion up to terms in $x^3$, and A1 for correctly obtaining the answer $x^2 + 7x - 6$ and also showing there is no term in $x^3$.]

[Allow the use of Maclaurin, giving M1A1 $\sqrt{}$ for $f(0) = -3$ and $f'(0) = 2$, A1 $\sqrt{}$ for $f''(0) = -3$, A1 $\sqrt{}$ for $f'''(0) = \frac{33}{2}$, and A1 for obtaining the given answer correctly (f.t. is on $A$, $B$, $C$ if used).]

10

(i) State $x$-coordinate of $A$ is 1  
B1

(ii) Use product or quotient rule  
M1
Obtain derivative in any correct form e.g. $-\frac{2\ln x}{x^3} + \frac{1}{x} \cdot \frac{1}{x^2}$  
A1
Equate derivative to zero and solve for $\ln x$  
M1
Obtain $x = e^\frac{1}{2}$ or equivalent (accept 1.65)  
A1
Obtain $y = \frac{1}{2e}$ or exact equivalent not involving $\ln$  
A1

[SR: if the quotient rule is misused, with a ‘reversed’ numerator or $x^2$ instead of $x^4$ in the denominator, award M0A0 but allow the following M1A1A1.]

(iii) Attempt integration by parts, going the correct way  
M1
Obtain $-\frac{\ln x}{x} + \int \frac{1}{x} \cdot \frac{1}{x} dx$ or equivalent  
A1
Obtain indefinite integral $-\frac{\ln x}{x} - \frac{1}{x}$  
A1
Use $x$-coordinate of $A$ and $e$ as limits, having integrated twice  
M1
Obtain exact answer $1 - \frac{2}{e}$, or equivalent  
A1

[If $u = \ln x$ is used, apply an analogous scheme to the result of the substitution.]
11 (i) **EITHER:** Obtain a vector in the plane e.g. \( \overrightarrow{PQ} = -3i + 4j + k \) \hspace{1cm} B1

Use scalar product to obtain a relevant equation in \( a, b, c \) e.g. \(-3a + 4b + c = 0\) or \(6a - 2b + c = 0\) or \(3a + 2b + 2c = 0\) \hspace{1cm} M1

State two correct equations in \( a, b, c \) \hspace{1cm} A1

Solve simultaneous equations to obtain one ratio e.g. \( a : b \) \hspace{1cm} M1

Obtain equation \( 2x + 3y - 6z = 8 \) or equivalent \hspace{1cm} A1

[The second M1 is also given if say \( c \) is given an arbitrary value and \( a \) or \( b \) is found. The following A1 is then given for finding the correct values of \( a \) and \( b \).]

**OR:** Substitute for \( P, Q, R \) in equation of plane and state 3 equations in \( a, b, c, d \) \hspace{1cm} B1

Eliminate one unknown, e.g. \( d \), entirely \hspace{1cm} M1

Obtain 2 equations in 3 unknowns \hspace{1cm} A1

Solve to obtain one ratio e.g. \( a : b \) \hspace{1cm} M1

Obtain equation \( 2x + 3y - 6z = 8 \) or equivalent \hspace{1cm} A1

[The first M1 is also given if say \( d \) is given an arbitrary value and two equations in two unknowns, e.g. \( a \) and \( b \), are obtained. The following A1 is for two correct equations. Solving to obtain one unknown earns the second M1 and the following A1 is for finding the correct values of \( a \) and \( b \).]

**OR:** Obtain a vector in the plane e.g. \( \overrightarrow{QR} = 6i - 2j + k \) \hspace{1cm} B1

Find a second vector in the plane and form correctly a 2-parameter equation for the plane \hspace{1cm} M1

Obtain equation in any correct form e.g. \( r = \lambda(-3i + 4j + k) + \mu(6i - 2j + k) + i - k \) \hspace{1cm} A1

State 3 equations in \( x, y, z, \lambda, \) and \( \mu \) \hspace{1cm} A1

Eliminate \( \lambda \) and \( \mu \) \hspace{1cm} M1

Obtain equation \( 2x + 3y - 6z = 8 \) or equivalent \hspace{1cm} A1

**OR:** Obtain a vector in the plane e.g. \( \overrightarrow{PR} = 3i + 2j + 2k \) \hspace{1cm} B1

Obtain a second vector in the plane and calculate the vector product of the two vectors, e.g. \((-3i + 4j + k) \times (3i + 2j + 2k)\) \hspace{1cm} M1

Obtain 2 correct components of the product \hspace{1cm} A1

Obtain correct product e.g. \( 6i + 9j - 18k \) or equivalent \hspace{1cm} A1

Substitute in \( 2x + 3y - 6z = d \) and find \( d \) or equivalent \hspace{1cm} M1

Obtain equation \( 2x + 3y - 6z = 8 \) or equivalent \hspace{1cm} A1
(ii) EITHER: State equation of $SN$ is $r = 3i + 5j - 6k + \lambda(2i + 3j - 6k)$ or equivalent $B1\sqrt{ }$

Express $x$, $y$, $z$ in terms of $\lambda$ e.g. $(3 + 2\lambda, 5 + 3\lambda, -6 - 6\lambda)$ $B1\sqrt{ }$

Substitute in the equation of the plane and solve for $\lambda$ $M1$

Obtain $\overrightarrow{ON} = i + 2j$, or equivalent $A1$

Carry out method for finding $SN$ $M1$

Show that $SN = 7$ correctly $A1$

OR: Letting $\overrightarrow{ON} = xi + yj + zk$, obtain two equations in $x$, $y$, $z$ by equating scalar product of $\overrightarrow{NS}$ with two of $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RP}$ to zero $B1\sqrt{ } + B1\sqrt{ }$

Using the plane equation as third equation, solve for $x$, $y$, and $z$ $M1$

Obtain $\overrightarrow{ON} = i + 2j$, or equivalent $A1$

Carry out method for finding $SN$ $M1$

Show that $SN = 7$ correctly $A1$

OR: Use Cartesian formula or scalar product of $\overrightarrow{PS}$ with a normal vector to find $SN$ $M1$

Obtain $SN = 7$ $A1$

State a unit normal $\hat{n}$ to the plane $B1\sqrt{ }$

Use $\overrightarrow{ON} = \overrightarrow{OS} \pm 7\hat{n}$ $M1$

Obtain an unsimplified expression e.g. $3i + 5j - 6k \pm 7(\frac{1}{7}i + \frac{5}{7}j - \frac{6}{7}k)$ $A1\sqrt{ }$

Obtain $\overrightarrow{ON} = i + 2j$, or equivalent, only $A1$
June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/04

MATHEMATICS
Paper 4 (Mechanics 1)
1. (i) \[ F = 13 \cos \alpha \]
Frictional component is 12 N
For resolving forces horizontally
M1 A1 2

(ii) \[ R = 1.1 \times 10 + 13 \sin \alpha \]
Normal component is 16 N
For resolving forces vertically (3 terms needed)
M1 A1 2

(iii) Coefficient of friction is 0.75
B1 ft 1

2. \[ X = 100 + 250 \cos 70^\circ \]
\[ Y = 300 - 250 \sin 70^\circ \]
\[ R^2 = 185.5^2 + 65.1^2 \]
\[ R = 197 \]

\[ \tan \alpha = 65.1/185.5 \]
\[ \alpha = 19.3^\circ \]
For using \( R^2 = X^2 + Y^2 \)
ft only if one B1 is scored or if the expressions for the candidate’s X and Y are those of the equilibrant
M1 A1 ft 6

OR
\[ X = 100 + 250 \sin 70^\circ \] and \[ Y = 300 - 250 \cos 70^\circ \]
( 334.9 and 214.5 )
B1 Method marks as scheme M1 M1

\[ R = 398 \text{ N and } \alpha = 32.6^\circ \]
\[ R = 316 \text{ ft} \]
\[ \alpha = 19.3^\circ \]
Magnitude of the resultant of two of the forces
Direction of the resultant of two of the forces
B1 B1 M1
For using the cosine rule to find \( R \)
A1 ft ft only if one B1 is scored
For using \( \sin \alpha = Y/X \)
A1 ft ft only if one B1 is scored

3. (i) Distance \( AC \) is 70 m
\[ 7 \times 10 - 4 \times 15 \]
Distance \( AB \) is 10 m
B1 M1 A1 3
For using \( |AB| = |AC| - |BC| \)

(ii) \[ x(m) \]
\[ t(s) \]
Graph consists of 3 connected straight line segments with, in order, positive, zero and negative slopes. \( x(t) \) is single valued and the graph contains the origin
M1
A1 ft 3
1st line segment appears steeper than the 3rd and the 3rd line segment does not terminate on the \( t \)-axis
Values of \( t \) (10, 15 and 30) and \( x \) (70, 70, 10) shown, or can be read without ambiguity from the scales
SR (max 1 out of 3 marks)
For first 2 segments correct B1
### 4

**(i)**

\[ KE = 0.2g(0.7) \]

Kinetic energy is 1.4 J

**M1**

**A1** 2

For using \( KE = PE \) lost and PE lost = \( mgh \)

**(ii)**

\[ R = 0.2 \times 10 \times \cos 16.3^\circ \]

\[ F = 0.288 \text{ N} \]

\[ WD = 0.72 \text{ J or } a = 1.36 \]

or resultant downward force = 0.272 N

\[ KE = 1.4 - 0.72 \]

or

\[ KE = \frac{1}{2} 0.2 (2 \times 1.36 \times 2.5) \] or

\[ 0.272 \times 2.5 \]

Kinetic energy is 0.68 J

**B1**

**B1 ft**

**B1 ft**

**M1**

**A1 ft** 5

From 0.15R (may be implied by subsequent exact value 0.72, 1.36 or 0.68)

From 2.5F or from 0.2a = 0.2 \times 10 \times (7/25) - F (may be implied by subsequent exact value 0.68)

For KE = PE lost – WD or

KE = \( \frac{1}{2} \) \( mv^2 \) and \( v^2 = 2as \) or

KE = resultant downward force \( \times 2.5 \)

### 5

**(i)**

\[ 10t^2 - 0.25t^4 \] \(+C\)

Expression is \( 10t^2 - 0.25t^4 - 36 \)

**M1**

**DM1**

**A1** 3

For integrating \( v \)

For including constant of integration and attempting to evaluate it

**(ii)**

Displacement is 60 m

**A1 ft** 1

Dependent on both M marks in (i); ft if there is not more than one error in \( s(t) \)

**(iii)**

\[ (t^2 - 36)(1 - 0.25t^2) = 0 \]

Roots of quadratic are 4, 36

\[ t = 2, 6 \]

**M1**

**A1**

**A1 ft** 3

For attempting to solve \( s = 0 \) (depends on both method marks in (i)) or \( \int_0^t v \text{ } dt = 36 \) (but not –36) for \( t^2 \) by factors or formula method

ft only from 3 term quadratic in \( t^2 \)

### 6

**(i)**

\[ DF - 400 = 1200 \times 0.5 \]

\[ 20000 = 1000v \]

\[ Speed = 20 \text{ ms}^{-1} \]

**M1**

**A1**

**M1**

**A1** 4

For using Newton’s 2\textsuperscript{nd} law (3 terms needed)

For using \( P = Fv \)

**(ii)**

\[ 20000/v - 400 = 0 \]

\[ v_{\text{max}} = 50 \text{ ms}^{-1} \]

**M1**

**A1** 2

For using \( P = Fv \) and Newton’s 2\textsuperscript{nd} law with \( a = 0 \) and \( F = 400 \)

AG

**(iii)**

\[ 20000 = \frac{1500000}{\Delta T} \]

or

distance = 1500 000/400 = 3750

and

time = 3750/50

Time taken is 75 s

**M1**

**A1** 2

For using \( P = \frac{\Delta W}{\Delta T} \) or for using ‘distance = work done/400’ and ‘time =distance/50’
7 (i) \[25 = 30t - 5t^2 \Rightarrow t^2 - 6t + 5 = 0 \Rightarrow \]
\[(t - 1)(t - 5) = 0\]
or
\[v^2 = 30^2 - 500; t_{up} = (20 - 0)/10\]
t = 1, 5 or \(t_{up} = 2\)
Time = 5 - 1 = 4 s or
Time = 2 \times 2 = 4 s    or    1 < t < 5
M1  For using \(25 = ut - \frac{1}{2}gt^2\) and
attempting to solve for \(t\)
or for using \(v^2 = u^2 - 2g(25)\) and
\(t_{up} = (v - 0)/g\)

(ii) \[s_1 = 30t - 5t^2\] and \(s_2 = 10t - 5t^2\)

\[30t - 10t = 25\]
t = 1.25
\[v_1 = 30 - 10 \times 1.25\] or
\[v_2 = 10 - 10 \times 1.25\]
or
\[v_1^2 = 30^2 - 2 \times 10(29.6875)\] or
\[v_2^2 = 10^2 - 2 \times 10(4.6875)\]
Velocities 17.5 ms\(^{-1}\) and -2.5 ms\(^{-1}\)
M1  For using \(s = ut - \frac{1}{2}gt^2\) for \(P_1\)
and \(P_2\)
M1  For using \(s_1 = s_2 + 25\) and
attempting to solve for \(t\)
A1 3

(iii) \[t_{up} = 3\]
\[3 - 1.25\]
Time is 1.75 s or 1.25 < t < 3

OR

(ii) \[v_1 = 30 - 10t, v_2 = 10 - 10t\]
\[v_1 - v_2 = 20\]
\[(30^2 - v_1^2) ÷ 20 = (10^2 - v_2^2) ÷ 20 + 25\]
\[v_1 - v_2 = 20, v_1^2 - v_2^2 = 300\]
Velocities are 17.5 ms\(^{-1}\) and -2.5 ms\(^{-1}\)
M1  For using \(v = u - gt\) (either
case) or for calculating \(s_1\) and
substituting into
\(v_1^2 = 30^2 - 2 \times 10s_1\) or
calculating \(s_2\) and substituting
into \(v_2^2 = 10^2 - 2 \times 10s_2\)
A1 5

(iii) \[0 = 17.5 - 10t\]
Time is 1.75 s or 1.25 < t < 3

SR (max 1 out of 3 marks)
0 = 17.5 + 10t
B1 ft

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Mechanics 2

1. For taking moments about the edge of the platform M1
   
   \[(75g \times 0.9 = 25g \times x + 10g \times 1.1) \text{ (3 term equation)}\]

   Two terms correct (ununsimplified) A1
   Comletely correct (ununsimplified) A1
   Distance \(MC = 3.16m\) A1

   **NB:** If moments taken about other points, the force of the platform on the 
   plank must be present at the edge of the platform for M1

2. (i) Evaluates \(\frac{2r \sin \alpha}{3\alpha} \times \cos \frac{\pi}{4}\) M1

   Obtains given answer correctly A1 2

   (ii) For taking moments about AB M1

   \(\{(5 \times 10 + \frac{1}{4} \pi 5^2)x = (5 \times 10) \times 5 + \frac{1}{4} \pi 5^2(10 + \frac{20}{3\pi})\}\)

   For the total area correct and the moment of the rectangle correct 
   (ununsimplified) A1
   For the moment of CDE correct (ununsimplified) A1
   Distance is 7.01 cm A1

3. For applying Newton’s 2nd law and using \(a = v \frac{dv}{dx}\) M1

   \(0.6v \frac{dv}{dx} = \frac{3}{x^4}\) A1

   For separating the variables and integrating M1

   \(0.3v^2 = \frac{3x^{-2}}{-2}\) (+C) A1 ft

   (ft omission of minus sign in line 2 only)

   For using \(= 0\) when \(x = 10\) M1

   \(v^2 = \frac{5}{x^2} - \frac{1}{20}\) (aef) A1 ft

   (ft wrong sign in line 4 only)

   Speed is \(\frac{\sqrt{3}}{2} \text{ ms}^{-1} (=0.866)\) A1 7
4 (i) Distance of the rod from the hinge is $\frac{2.4}{2.5}(0.7)$ or $0.7\cos16.26^\circ$ (=0.672) B1

[May be implied in moment equation]

For taking moments about the hinge (3 term equation) M1

$0.672F = 68 \times 1.2 + 750 \times 2.4$ A1 ft

Force is 2800 N A1 4

(ii) $X = 784$ (ft for 0.28F) B1 ft

For resolving vertically (4 term equation) M1

$Y = 1870$ (ft for 0.96F – 818) A1 ft

SR: For use of 680 N for weight of the beam: (i) B1, M1, A0. In (ii) ft 680, so 3/3 possible.

5 (i) For using $EPE = \frac{\lambda x^2}{2L}$ M1

$EPE\ gain = 2 \left(\frac{200x^2}{2 \times 4}\right) (=50x^2)$ A1

GPE loss = $10g (4 + x)$ B1

For using the principle of conservation of energy to form an equation M1

containing $EPE$, $GPE$ and KE terms

[$\frac{1}{2}10^2 + 50x^2 = 10g (4 + x)$]

Given answer obtained correctly A1 5

ALTERNATIVE METHOD:

$T = \frac{200x}{4}$ B1

$100 - 2 \left(\frac{200x}{4}\right) = 10v \frac{dv}{dx}$ M1

$\frac{1}{2}v^2 = 10x - 5x^2$ (+C) A1

Use $x = 0, \quad v^2 = 8g$ M1

$= 10(8 + 2x - x^2)$ A1

(ii) For using $= 0$ and factorizing or using formula method for solving M1

$x = 4$ (only) A1 2
6 (i) \[2 = VT\sin35° - 5T^2 \quad \text{or} \quad 2 = 25\tan35° - \frac{25^2 \times 10}{2V^2 \cos^2 35°}\] B1

\[25 = VT\cos35°\] B1

For obtaining \(V^2\) or \(T^2\) in \(AV^2 = B\) or \(CT^2 = D\) form where \(A,B,C,D\) are numerical M1

\[[(25\tan35° - 2)\cos^235°]V^2 = 3125 \quad \text{(aef)} \quad \text{or} \quad 5T^2 = 25\tan35° - 2 \quad \text{(aef)}\]

\[V = 17.3 \quad \text{or} \quad T = 1.76\] A1

\[T = 1.76 \quad \text{or} \quad V = 17.3 \quad \text{(ft} \quad VT = 30.519365)\] B1 ft 5

(ii) For using \(v = V\sin35° - gT\) \quad \text{(must be component of V for M1)} M1

\[\dot{y}_M = (9.94 - 17.61 = -7.67) < 0 \rightarrow \text{moving downwards} \quad \text{A1 ft}\]

(ft on \(V\) and \(T\))

For using \(m^2 = (V\cos35°)^2 + \dot{y}_M^2\) M1

\((m^2 = (14.20)^2 + (-7.67)^2) \quad \text{or}\)

For using the principle of conservation of energy

\[(\frac{1}{2}m(v^2_{M} - 17.3^2) = -mg \times 2)\]

\[m = 16.1 \text{ ms}^{-1}\] A1 4

LINES 1 AND 2 ALTERNATIVE METHODS

EITHER Compare 25 with \(\frac{1}{2}R\left(\frac{v^2 \sin 70°}{g}\right)\) M1

\[25 > 14.1 \rightarrow \text{moving downwards} \quad \text{A1}\]

OR Compare 1.76 with time to greatest height \(\left(\frac{V\sin35°}{g}\right)\) M1

\[1.76 > 0.994 \rightarrow \text{moving downwards} \quad \text{A1}\]

OR \(\frac{dy}{dx} = \tan35° - \frac{g \cdot 10}{V^2 \cos^2 35°}(-0.54) \quad \text{used}\) M1

As \(\tan 3\) is negative \(→ \text{moving downwards}\) A1
7 (i) \( T \cos 60^\circ = 0.5g \)  \((T = 10)\)

For applying Newton’s 2\textsuperscript{nd} law horizontally and using \( a = \frac{v^2}{r} \)  

(must be a component of \( T \) for M1)

\( T \sin 60^\circ = \frac{0.5v^2}{0.15 \sin 60^\circ} \)  (for an equation in \( V^2 \))

For substituting for \( T \)  

\( = 1.5 \)

ALTERNATIVELY:

\( a = \frac{v^2}{0.15 \sin 60^\circ} \)  

For applying Newton’s 2\textsuperscript{nd} law perpendicular to the string  

\( 0.5g \cos 30^\circ = 0.5(\cos 60^\circ) \)  

For substituting for \( a \)

\( (5 \cos 30^\circ = 0.5 \times 0.15 \tan 60^\circ) \)  (for an equation in \( V^2 \))

\( = 1.5 \)

(ii) (a) \( T \sin 45^\circ = \frac{0.5(0.9)^2}{0.15 \sin 45^\circ} \)

Tension is 5.4 N

(b) For resolving forces vertically

\( 5.4 \cos 45^\circ + R = 0.5g \)

Force is 1.18 N
June 2004

GCE A AND AS LEVEL
AICE

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SYLLABUS/COMPONENT: 9709/06, 0390/06

MATHEMATICS
Paper 6 (Probability and Statistics 1)
1 (i) \[ \bar{x}_A = 139 \quad (138.75) \]
\[ \sigma_A = 83.1 \]

(ii) team B
smaller standard deviation

For the mean
For the sd

2 (i) axes and labels
points

For correct uniform scales and labels on both axes, accept Frequency, %CF.
Number of people, allow axes reversed, allow halves
For 3 correct points
All points correct and reasonable graph incl straight lines

(ii) accept 60 – 70 for straight lines
40 – 70 for curve

For subtracting from 640 can be implied
For correct answer, reasonably compatible with graph

3 (i) | x | P(X = x) |
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For 36 in the uncancelled denominator somewhere, accept decimals eg 0.305 recurring or 0.306 etc
For 3 correct probabilities
All correct

(ii) \[ E(X) = 1 \times \frac{11}{36} + 2 \times \frac{9}{36} + 3 \times \frac{7}{36} + 4 \times \frac{5}{36} + 5 \times \frac{3}{36} + 6 \times \frac{1}{36} = \frac{91}{36} \]

For calculation of \( \sum xP \) where all probs < 1

4 (i) \[ z = \frac{350 - 450}{120} = -0.833 \]
% small = 1 – 0.7975 = 0.2025 or 20.25%

For standardising accept 120 or \( \sqrt{120} \), no cc
For correct z value, + or -, accept 0.83
For answer rounding to 0.202 or 0.203

(ii) \[ 0.7975 \div 2 = 0.39875 \] each
\[ \Phi z = 0.60125 \]
\[ z = 0.257 \]
\[ x = 120 \times 0.257 + 450 = 481 \]

For dividing their remainder by 2
For adding their above two probs together or sub from 1
For finding the z corresponding to their probability
For converting to x from a z value
For answer, rounding to 481
### Question 5 (a) (i)

\[3 \times 5 \times 3 \times 2 \text{ or } 3C_1 \times 5C_1 \times 3C_1 \times 2 \]

\[= 90\]

**Marks:**
- M1: For multiplying \(3 \times 5 \times 3\)
- A1: For correct answer

#### (ii)

\[(3 \times 5 \times 2) + (3 \times 3) + (5 \times 2 \times 3)\]

\[= 69\]

**Marks:**
- M1: For summing options that show S&M, S&D, M&D
- M1: \(3 \times 5 \times a + 3 \times 3 \times b + 5 \times 3 \times c \) seen for integers \(a, b, c\)
- A1: For correct answer

### Question 5 (b)

\[\binom{14}{5} \times \binom{9}{5} \times \binom{4}{4} \text{ or equivalent} \]

\[= 252252\]

**Marks:**
- M1: For using combinations not all \(\binom{14}{5}\)
- M1: For multiplying choices for two or three groups
- A1: For correct answer

- NB: \(14!/5!5!4!\) scores M2 and A1 if correct answer

### Question 6 (i)

- Top branch: 0.65, 0.9, 0.1
- Bottom branch: 0.35, 0.8, 0.2

**Marks:**
- B1: For top branches correct (0.65, 0.9, 0.1)
- B1: For bottom branches correct (0.35, 0.8, 0.2)
- B1: For win/lose option after 2nd in (0.6, 0.4)
- B1: For all labels including final lose at end of bottom branch

### Question 6 (ii)

\[0.65 \times 0.1 + 0.35 \times 0.8 \times 0.4 + 0.35 \times 2\]

\[= 0.247\]

**Marks:**
- M1: For evaluating 1st in and lose seen
- M1: For 1st out 2nd in lose, or 1st out 2nd out lose
- A1: For correct answer

### Question 6 (iii)

\[\frac{0.65 \times 0.1}{0.247} \]

\[= 0.263 \text{ (}= \frac{5}{19})\]

**Marks:**
- M1: For dividing their 1st in and lose by their answer to (ii)
- A1ft: For correct answer, ft only on \(0.65 \times 0.1/\text{their (ii)}\)

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### Question 7

**Part (i)**

Given: $P(0) = (0.8)^{15}$

- $P(0) = 0.03518$ (for correct numerical expression for $P(0)$)

- $P(1) = \binom{15}{1} \times (0.2) \times (0.8)^{14} = 0.1319$ (for correct numerical expression for $P(1)$ or $P(2)$)

- $P(2) = \binom{15}{2} \times (0.2)^2 \times (0.8)^{13} = 0.2309$ (for answer rounding to 0.398)

- $P(X \leq 2) = 0.398$

**Part (ii)**

1. $1 - (0.8)^n \geq 0.85$
2. $0.15 \geq (0.8)^n$

- $n = 9$

**Part (iii)**

Given: $\mu = 1600 \times 0.2 = 320$, $\sigma^2 = 1600 \times 0.2 \times 0.8 = 256$

- $P(X \geq 290) \text{ or } P(X < 350)$

- $P(X \geq 290) = 1 - \Phi\left(\frac{289.5 - 320}{\sqrt{256}}\right) = 1 - \Phi(-1.906)$

- $P(X < 350) = \Phi(1.906) = 0.972$ (for both mean and variance correct)

- For standardising, with or without cc, must have $\sqrt{\sigma^2}$ on denom

- For use of continuity correction 289.5 or 290.5

- For finding an area > 0.5 from their $z$

- For answer rounding to 0.972
<table>
<thead>
<tr>
<th>MARK SCHEME</th>
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</thead>
<tbody>
<tr>
<td>MAXIMUM MARK: 50</td>
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</tbody>
</table>

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)
1 (i) $H_0: \mu = 15$ or $p = 0.25$
$H_1: \mu > 15$ or $p > 0.25$

(ii) Test statistic
$z = \frac{93.81 - 15}{75.0 \times 0.25 \times 0.75} = 1.938$

OR test statistic
$z = \frac{22/60 - 0.5/60 - 15/60}{0.25 \times 0.75} = 1.938$

CV $z = 1.645$

In CR Claim justified

<table>
<thead>
<tr>
<th>Mark</th>
<th>Scheme</th>
<th>Syllabus</th>
<th>Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (i)</td>
<td>B1</td>
<td>1</td>
<td>For $H_0$ and $H_1$, correct</td>
</tr>
<tr>
<td>(ii)</td>
<td>M1</td>
<td>For attempt at standardising with or without cc, must have $\sqrt{\cdot}$ something with 60 in on the denom</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For 1.94 (1.938)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1ft</td>
<td>For comparing with 1.645 or 1.96 if 2-tailed, signs consistent, or comparing areas to 5% For correct answer(ft only for correct one-tail test)</td>
<td></td>
</tr>
</tbody>
</table>

2 (i) Mean = $3.5 + 2.9 + 3.1 = 9.5$
Var = $0.3^2 + 0.25^2 + 0.35^2 = 0.275$
St dev = 0.524

(ii) $z = \frac{9 - 9.5}{\sqrt{\text{their var}}} = -1.907$

or $z = \frac{36-38}{\sqrt{4 \times \text{their var}}} = -1.907$

$\Phi(1.907) = 0.9717 = 0.972$

<table>
<thead>
<tr>
<th>Mark</th>
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</tr>
</thead>
<tbody>
<tr>
<td>2 (i)</td>
<td>B1</td>
<td>9.5 as final answer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>For summing three squared deviations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For correct answer</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>M1</td>
<td>For standardising, no cc</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>For $\sqrt{\text{their var}}$ or $\sqrt{4 \times \text{their var}}$ in denom - no 'mixed' methods.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For correct answer</td>
<td></td>
</tr>
</tbody>
</table>

3 (i) $E(2X-3Y) = 2E(X) - 3E(Y) = 16 - 18 = -2$

(ii) Var $(2X-3Y) = 4\text{Var} (X) + 9\text{Var} (Y)$
$= 19.2 + 54$
$= 73.2$

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>3 (i)</td>
<td>M1</td>
<td>For multiplying by 2 and 3 resp and subt</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For correct answer</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>B1</td>
<td>For use of $\text{var} (Y) = 6$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>For squaring 3 and 2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>For adding variances (and nothing else)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For correct final answer</td>
<td></td>
</tr>
</tbody>
</table>

4 (i) $\bar{x} = 375.3$
$\sigma^2_{n-1} = 8.29$

<table>
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<tbody>
<tr>
<td>4 (i)</td>
<td>B1</td>
<td>For correct mean (3.s.f)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>For legit method involving $n-1$, can be implied</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For correct answer</td>
<td></td>
</tr>
</tbody>
</table>

(ii) $p = 0.19$ or equiv.

$0.19 \pm 2.055 \times \sqrt{\frac{0.19 \times 0.81}{200}}$

$0.133 < p < 0.247$

<table>
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</thead>
<tbody>
<tr>
<td>4 (ii)</td>
<td>B1</td>
<td>For correct $p$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M1</td>
<td>For correct form $p \pm z \times \sqrt{\frac{pq}{n}}$ either/both sides</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>For $z = 2.054$ or 2.055</td>
<td></td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>For correct answer</td>
<td></td>
</tr>
</tbody>
</table>
### Question 5

#### Part (i)

\[ \frac{c - 54}{3.1/\sqrt{10}} = -1.282 \]

\[ c = 54 - 1.282 \times \frac{3.1}{\sqrt{10}} = 52.74 \]

**Mark Scheme**

- **B1**: For + or – 1.282 seen
- **M1**: For equality/inequality with their \( z \) \((\pm)\) (must have used tables), no \( \sqrt{10} \) needed (c can be numerical)
- **A1**: For correct expression (c can be numerical, but signs must be consistent)
- **A1**: For correct GIVEN answer. No errors seen.

#### Part (ii)

\[ P(x > 52.74) = 1 - \Phi \left( \frac{52.74 - 51.5}{3.1/\sqrt{10}} \right) \]

\[ = 1 - \Phi(1.265) = 1 - 0.8971 \]

\[ = 0.103 \text{ or } 0.102 \]

**Mark Scheme**

- **B1**: For identifying the outcome for a type II error
- **M1**: For standardising, no \( \sqrt{10} \) needed
- **A1**: For \( \pm 1.265 \) (accept 1.26-1.27)

### Question 6

#### Part (i)

\[ P(5) = e^{-6} \times \frac{6^5}{5!} = 0.161 \]

**Mark Scheme**

- **M1**: For an attempted Poisson \( P(5) \) calculation, any mean
- **A1**: For correct answer

#### Part (ii)

\[ P(X \geq 2) = 1 - P(0) - P(1) \]

\[ = 1 - e^{-1.6}(1 + 1.6) \]

\[ = 0.475 \]

**Mark Scheme**

- **B1**: For \( \mu = 1.6 \), evaluated in a Poisson prob
- **M1**: For \( 1 - P(0) - P(1) \) or \( 1 - P(0) - P(1) - P(2) \)
- **A1**: For correct answer

#### Part (iii)

\[ P(1 \text{ then } 4|5) = \frac{(e^{-3} \times 3) \times (e^{-3} \times \frac{3^4}{4!})}{e^{-6} \times \frac{6^5}{5!}} \]

\[ = 0.156 \text{ or } 5/32 \]

**Mark Scheme**

- **M1**: For multiplying \( P(1) \) by \( P(4) \) any (consistent) mean
- **M1**: For dividing by \( P(5) \) any mean
- **A1**: For correct answer

### Question 7

#### Part (i)

\[ \int_0^{25-t^2} c \, dt = 1 \]

\[ \left[ \frac{25r^2}{2} - \frac{r^4}{4} \right]_0^5 = 1 \]

\[ \frac{625}{2} - \frac{625}{4} = 1 \Rightarrow c = \frac{4}{625} \]

**Mark Scheme**

- **M1**: For equating to 1 and a sensible attempt to integrate
- **A1**: For correct integration and correct limits
- **A1**: For given answer correctly obtained

#### Part (ii)

\[ \int_2^4 cr(25-t^2) \, dt = \left[ \frac{25cr^2}{2} - \frac{cr^4}{4} \right]_2^{136} - c[46] \]

\[ = \frac{72}{125} \text{ (0.576)} \]

**Mark Scheme**

- **M1**: For attempting to integrate \( f(t) \) between 2 and 4 (or attempt 2 and 4)
- **M1**: For subtracting their value when \( t = 2 \) from their value when \( t = 4 \)
- **A1**: For correct answer

#### Part (iii)

\[ \int_0^5 cr(25-t^2) \, dt = \left[ \frac{4}{625} \times \frac{25r^3}{3} - \frac{4}{625} \times \frac{r^5}{5} \right]_0^5 \]

\[ = \frac{8}{3} \]

**Mark Scheme**

- **M1**: For attempting to integrate \( ff(t) \), no limits needed
- **A1**: For correct integrand can have \( c \) (or their \( c \))
- **M1**: For subtracting their value when \( t=0 \) from their value when \( t=5 \)
- **A1**: For correct answer