CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS
PAPER 2  Pure Mathematics 2 (P2)

MAY/JUNE SESSION 2002
1 hour 15 minutes

Additional materials:
Answer paper
Graph paper
List of Formulae (MF9)

TIME 1 hour 15 minutes

INSTRUCTIONS TO CANDIDATES
Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
1 Solve the inequality \(|x + 2| < |5 - 2x| \).

2 The cubic polynomial \(3x^3 + ax^2 - 2x - 8\) is denoted by \(f(x)\).

(i) Given that \((x + 2)\) is a factor of \(f(x)\), find the value of \(a\).

(ii) When \(a\) has this value, factorise \(f(x)\) completely.

3 Two variable quantities \(x\) and \(y\) are related by the equation
   \[ y = Ax^n, \]
   where \(A\) and \(n\) are constants.

   ![Graph](image)

   When a graph is plotted showing values of \(\ln y\) on the vertical axis and values of \(\ln x\) on the horizontal axis, the points lie on a straight line. This line crosses the vertical axis at the point \((0, 2.3)\) and also passes through the point \((4.0, 1.7)\), as shown in the diagram. Find the values of \(A\) and \(n\).

4 (i) Express \(3 \cos \theta + 2 \sin \theta\) in the form \(R \cos(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), stating the exact value of \(R\) and giving the value of \(\alpha\) correct to 1 decimal place.

(ii) Solve the equation
    \[ 3 \cos \theta + 2 \sin \theta = 3.5, \]
    giving all solutions in the interval \(0^\circ \leq \theta \leq 180^\circ\).

(iii) The graph of \(y = 3 \cos \theta + 2 \sin \theta\), for \(0^\circ \leq \theta \leq 180^\circ\), has one stationary point. State the coordinates of this point.
The diagram shows the curve \( y = 2x e^{-x} \) and its maximum point \( P \). Each of the two points \( Q \) and \( R \) on the curve has \( y \)-coordinate equal to \( \frac{1}{2} \).

(i) Find the exact coordinates of \( P \). \[ 4 \]

(ii) Show that the \( x \)-coordinates of \( Q \) and \( R \) satisfy the equation

\[ x = \frac{1}{2} e^x. \] \[ 1 \]

(iii) Use the iterative formula

\[ x_{n+1} = \frac{1}{4} e^{x_n}, \]

with initial value \( x_1 = 0 \), to find the \( x \)-coordinate of \( Q \) correct to 2 decimal places, showing the value of each approximation that you calculate. \[ 3 \]

6 (a) (i) Show that \( \int_0^{\frac{1}{x}} \cos 2x \, dx = \frac{1}{2} \). \[ 2 \]

(ii) By using an appropriate trigonometrical identity, find the exact value of \( \int_0^{\frac{1}{x}} \sin^2 x \, dx \). \[ 3 \]

(b) (i) Use the trapezium rule with 2 intervals to estimate the value of \( \int_0^{\frac{1}{x}} \sec x \, dx \), giving your answer correct to 2 significant figures. \[ 3 \]

(ii) Determine, by sketching the appropriate part of the graph of \( y = \sec x \), whether the trapezium rule gives an under-estimate or an over-estimate of the true value. \[ 2 \]

7 The parametric equations of a curve are

\[ x = t + 2 \ln t, \quad y = 2t - \ln t, \]

where \( t \) takes all positive values.

(i) Express \( \frac{dy}{dx} \) in terms of \( t \). \[ 3 \]

(ii) Find the equation of the tangent to the curve at the point where \( t = 1 \). \[ 3 \]

(iii) The curve has one stationary point. Show that the \( y \)-coordinate of this point is \( 1 + \ln 2 \) and determine whether this point is a maximum or a minimum. \[ 4 \]