1 Solve the equation \( \ln(x^2 + 4) = 2 \ln x + \ln 4 \), giving your answer in an exact form. \[3\]

2 Express the equation \( \tan(\theta + 45^\circ) - 2 \tan(\theta - 45^\circ) = 4 \) as a quadratic equation in \( \tan \theta \). Hence solve this equation for \( 0^\circ \leq \theta \leq 180^\circ \). \[6\]

3 The equation \( x^5 - 3x^3 + x^2 - 4 = 0 \) has one positive root.
   (i) Verify by calculation that this root lies between 1 and 2. \[2\]
   (ii) Show that the equation can be rearranged in the form
   \[ x = \sqrt[3]{3x + \frac{4}{x^2} - 1}. \] \[1\]
   (iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. \[3\]

4 The polynomial \( 4x^3 + ax + 2 \), where \( a \) is a constant, is denoted by \( p(x) \). It is given that \( (2x + 1) \) is a factor of \( p(x) \).
   (i) Find the value of \( a \). \[2\]
   (ii) When \( a \) has this value,
       (a) factorise \( p(x) \), \[2\]
       (b) solve the inequality \( p(x) > 0 \), justifying your answer. \[3\]

5 Let \( I = \int_0^1 \frac{9}{(3 + x^2)^2} \, dx \).
   (i) Using the substitution \( x = (\sqrt{3}) \tan \theta \), show that \( I = \sqrt{3} \int_0^{\frac{\pi}{6}} \cos^2 \theta \, d\theta \). \[3\]
   (ii) Hence find the exact value of \( I \). \[4\]

6 A curve has equation
   \[ \sin y \ln x = x - 2 \sin y, \]
   for \( -\frac{1}{2} \pi \leq y \leq \frac{1}{2} \pi \).
   (i) Find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \). \[5\]
   (ii) Hence find the exact \( x \)-coordinate of the point on the curve at which the tangent is parallel to the \( x \)-axis. \[3\]
The variables \( x \) and \( y \) satisfy the differential equation
\[
\frac{dy}{dx} = xe^{x+y},
\]
and it is given that \( y = 0 \) when \( x = 0 \).

(i) Solve the differential equation and obtain an expression for \( y \) in terms of \( x \). [7]

(ii) Explain briefly why \( x \) can only take values less than 1. [1]

The line \( l \) has equation \( r = \left( \begin{array}{c} 1 \\ 2 \\ -1 \end{array} \right) + \lambda \left( \begin{array}{c} 2 \\ 1 \\ 3 \end{array} \right) \). The plane \( p \) has equation \( r \cdot \left( \begin{array}{c} 2 \\ -1 \\ -1 \end{array} \right) = 6 \).

(i) Show that \( l \) is parallel to \( p \). [3]

(ii) A line \( m \) lies in the plane \( p \) and is perpendicular to \( l \). The line \( m \) passes through the point with coordinates (5, 3, 1). Find a vector equation for \( m \). [6]

Let \( f(x) = \frac{3x^3 + 6x - 8}{x(x^2 + 2)} \).

(i) Express \( f(x) \) in the form \( A + \frac{B}{x} + \frac{Cx + D}{x^2 + 2} \). [5]

(ii) Show that \( \int_{1}^{2} f(x) \, dx = 3 - \ln 4 \). [5]

(a) Find the complex number \( z \) satisfying the equation \( z^* + 1 = 2iz \), where \( z^* \) denotes the complex conjugate of \( z \). Give your answer in the form \( x + iy \), where \( x \) and \( y \) are real. [5]

(b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities \( |z + 1 - 3i| \leq 1 \) and \( \text{Im} \, z \geq 3 \), where \( \text{Im} \, z \) denotes the imaginary part of \( z \). [4]

(ii) Determine the difference between the greatest and least values of \( \text{arg} \, z \) for points lying in this region. [2]